AD-773 861

DEVELOPMENT AND APPLICATIONS OF THEORETICAL METHODS FOR EVALUATING STABILITY OF OPENINGS IN ROCK

Woodward-Lundgren and Associates

Prepared for

Bureau of Mines

Advanced Research Projects Agency

December 1973

DISTRIBUTED BY:

NTIS

National Technical Information Service
U. S. DEPARTMENT OF COMMERCE

ia AD 773 861

UNCLASSIFIED
Security Classification

3200.8 (Att 1 to Encl 1)
Mar 7, 66

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
Woodward-Lundgren & Associates
2730 Adeline Street
Oakland, California 94607

2a. REPORT SECURITY CLASSIFICATION
UNCLASSIFIED

2b. GROUP

3. REPORT TITLE
DEVELOPMENT AND APPLICATIONS OF THEORETICAL METHODS FOR EVALUATING STABILITY OF OPENINGS IN ROCK

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Final Report

5. AUTHOR(S) (First name, middle initial, last name)
Chin-Yung Chang and Keshavan Nair

6. REPORT DATE
December, 1973

7a. TOTAL NO. OF PAGES
209

7b. NO. OF REFS
46

8a. CONTRACT OR GRANT NO.
HO220038

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited.

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Advanced Research Projects Agency
Washington, D.C. 20301

13. ABSTRACT

The purposes of this study are: (1) To incorporate into the finite element computer program for plane strain analyses, with capabilities to perform joint perturbation, no tension and elasto-plastic analyses developed under Contract No. HO210046, the capability to model and analyze: (a) structural support schemes used in the construction and design of tunnels, and (b) excavation techniques and construction and excavation sequences in underground construction. (2) To evaluate the analytical method developed in (1) by a study of case histories. This report discusses (i) the available information on excavation techniques, construction and excavation sequences, the mechanisms of ground support and the current design techniques for support schemes; (ii) formulation of the computational models on the basis of the review in (1); (iii) modification of the computer program to incorporate the computational models; and (iv) analysis of well-documented model tests and case histories of underground openings which include the analysis of model tests on lined and unlined openings in jointed rock, and the analysis of Tumut 1 Underground Power Station and a rock tunnel of Washington, D.C. Subway (METRO).

DD FORM 1 NOV 65 1473

UNCLASSIFIED
Security Classification

208

ib

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Underground opening						
No tension analysis						
Joint perturbation analysis						
Elasto-plastic analysis						
Support schemes						
Excavation techniques						
Construction and excavation sequence						
Tunnel design						
Excavation simulation						
Plane strain						

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FINAL REPORT

March 14, 1972 - December 14, 1973

ARPA Order Number: 1579, Amend. 3

Program Code Number: 2F10

Name of Contractor:
Woodward-Lundgren & Associates

Effective Date of Contract:
March 14, 1972

Contract Expiration Date:
December 14, 1973

Amount of Contract: $78,920

Contract Number: HO220038

Principal Investigators:
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Telephone Number (415)444-1256

Short Title of Work:
Development and Applications of Theoretical Methods for Evaluating Stability of Openings in Rock

This research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Bureau of Mines under Contract Number HO220038.

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TECHNICAL REPORT SUMMARY

CONTRACT OBJECTIVES

The objectives of this contract are:

(1) To incorporate into the finite element computer program for plane strain analysis, with capabilities to perform joint perturbation, no tension and elasto-plastic analyses developed under Contract No. H0210046, the capability to model and analyze: (a) structural support schemes used in the construction and design of tunnels, and (b) typical excavation sequences utilized in underground construction, and

(2) To evaluate the analytical method (computer program) developed in (1) by a study of case histories.

GENERAL APPROACH AND TECHNICAL RESULTS

The approach to this study can be divided into two phases:

I. On the basis of available information on excavation techniques, construction and excavation sequence, mechanisms of ground support and current design techniques for support systems, formulate computational models and modify the existing computer program to incorporate these computational models, and

II. Analyze case histories and compare predicted and measured performance.

(I) Formulation of Computational Models and Modifications of Existing Computer Program

On the basis of the review of the available information, the following modelling concepts were developed:

(1) Excavation Techniques - It is known that excavation of an opening creates some disturbance in a rock mass surrounding the opening. Depending upon the excavation technique, e.g., drilling and blasting, smooth wall blasting or boring machine, rock conditions and time of installation of support systems, the zones of loosening and fracturing and the depths of overbreak around the opening will be different. Zones of disturbance may be estimated on the basis of experience at locations with similar geologic conditions and excavation methods or determined by seismic refraction surveys in the field.

The essential features that have to be modelled in simulating the effects of excavation techniques are the following:

(i) The stress-free excavation face. It has been shown that excavation may be simulated in the finite element method by applying stresses to the boundary exposed by excavation so that there is no resultant stress on the excavation face. A similar technique was used in this study.

(ii) The disturbed zone in the vicinity of the excavation. This zone can be modelled by assuming a lower modulus for the material in the zone or by assuming that the material is incapable of carrying any tensile stress. Both these techniques were utilized in this study.

(2) Construction and Excavation Sequence - The essential features to be modelled and the basic concepts in modelling them are described below:

(i) The time sequence of construction, including installation of supports. Because of the limitations that only time-independent material properties can be included in the program, the time sequence of construction will be modelled in accordance with the following two-stage analyses:

(a) An initial analysis prior to any support installation will be conducted.

(b) A subsequent analysis will be conducted with the support system installation treating the results of the analysis in (a) as the initial condition. In a practical problem, it will be necessary to bracket possible initial conditions.

(ii) The excavation sequence. The opening goes through many shapes before reaching the final shape. If the problem could be treated as linear elastic, the final stress distribution would be independent of the excavation sequence; for non-linear problems, it is necessary to consider the sequence. Excavation sequence will be simulated by removing those elements that will be excavated and ensuring that the excavation face is stress free.

(3) Support Systems - This development is based on considering the interaction of the support and the surrounding rock mass. The three basic support systems considered in this study, (i) steel sets, (ii) rock bolts, and (iii) shotcrete liners, are discussed separately.

(i) Steel Sets - A series of beam elements which are capable of carrying both bending and axial stresses may be used to idealize a steel set. The support-rock connections i.e., blockings, may be idealized by a one-dimensional or a regular element if the connections are to transfer only axial forces or both axial and shear forces. As described previously, this study is confined to analysis of plane problems; and, thus, both the opening and its support system are to be idealized as plane strain problems. It is proposed that the sets along some length of the tunnel be idealized by a continuous support with a section modulus equivalent to the average section modulus of the sets. The blockings are assumed to be continuous along the length of the tunnel.

(ii) Rock Bolts - Because of difficulties associated with analysis of a rock bolt system i.e., the three-dimensional aspect, the interaction of each rock bolt with the rock cannot be modelled in this study. The following approximations are proposed to idealize the rock bolt support system:

(a) To increase the stiffness of the rock mass in the immediate vicinity of rock bolts to account for the presence of rock bolts and grouted rock bolts.

(b) To approximate the effects of tensioned bolts on the rock mass by applying a set of opposite concentrated loads at the anchor and bearing plate. Each concentrated load is considered to be an equivalent line load along the tunnel axis to represent a row of rock bolts. The

magnitude of the line load is determined by the bolt tension and the spacing of bolts.

(c) Untensioned grouted rock bolts may be idealized as one-dimensional bar elements with material properties similar to those of rock bolts.

(iii) Shotcrete or Concrete Linings - Shotcrete or concrete linings may be idealized as a plane strain structure. Grouting or back packing behind the lining may be modelled in the analysis with materials of different stiffness.

Before modifications were made for the present contract, two improvements were incorporated into the program. These were (i) utilization of elasto-plastic stress-strain relationship to compute the axial strain, and (ii) updating the element stiffness at each load increment to improve convergence.

Several example problems were solved using the modified computer program, and the results compared when possible with results published by other investigators.

(II) <u>Evaluation of Analytical Method (Computer Program) - Case History Studies</u>

To illustrate the use and evaluate the capabilities of the computer program model developed in (I) well documented case histories on the performance of underground openings were analyzed. These covered a range of conditions to illustrate the wide applicability of the program. Computed performance was compared with observed performance.

The cases analyzed are described below.

Model Tests - (a) Lined and (b) Unlined Openings

The model tests conducted by Hendron et al. (1972) on lined openings in jointed rock were analyzed. The model has a cu_ed opening of 4 inches in diameter and a 0.035-inch-thick aluminum liner and was constructed with a 2-inch joint spacing in two mutually perpendicular directions at 45° to the principal loading directions. The model was tested at a principal stress ratio, $\sigma_H/\sigma_V = 2/3$ to a maximum vertical model pressure of 1300 psi under plane strain conditions and was instrumented with eight pairs of buried extensometers and six diametrical extensometers in the tunnel liner to measure the deformability of the jointed model as well as the movements around the opening. Both the joints and the liner were modelled in the analysis.

The Tumut I Underground Power Station

The power station is situated under the lower part of the very steep eastern wall of the Tumut Valley in the Snowy Mountains of Southeast Australia. It is located about 1100 feet vertically below the ground surface, 1200 feet in from the river, and 150 feet below the level of the river bed. The machine hall is 306 feet in length, 44 feet in maximum width, and 104 feet in maximum height. The machine hall excavation was made in several stages. After the pilot tunnel was driven, the roof section of the machine hall was excavated to full width, systematical rock bolts and permanent concrete ribs installed. Following this, the main body of the machine hall was excavated by quarrying methods. The vertical walls and roof were systematically rock-bolted as soon as they were exposed. The behavior of the rock mass around the machine hall was observed during construction by strain measurements in many of the reinforced concrete arch ribs and measurements of the horizontal movements of points at the ends of the concrete ribs and on the rock walls, and angular rotation of points on the reinforced concrete abutment

beams and on the rock walls. A simplified excavation sequence and the influence of rock bolts and concrete ribs were included in the analysis. Two 'faults' which intersect the machine hall were also modelled in the analysis.

Rock Tunnel Washington D.C. METRO

The rock tunnel analyzed was driven through a foliated rock-schistose gneiss of quartz-mica composition. Average rock quality, defined as the RQD of the rock cores, ranges between fair to good, except in the shear zones where rock quality is poor to very poor. The geologic features present at the tunnel consist of four highly continuous, smooth, planar joint sets and eight major shear zones. The major shear zones and two of the joint sets are subparallel to rock foliation and strike within 10° of the axis of the tunnel. The tunnel was excavated in several stages together with installation of shotcrete, grouted rock bolts and steel ribs. Rock movements were monitored by a series of multiple position extensometer during excavation. Rock reinforcement, joints and a simplified excavation sequence were utilized in the analysis.

Results

Verification of the program in the strictest sense was not possible because in all the cases analyzed there was insufficient direct information to model all significant aspects of the problem. The major lack of information was found to be with respect to geologic discontinuities. It is believed that this will be true in most practical problems. Analysis of the case histories has shown that it is possible by means of a parametric study to select appropriate properties of geologic discontinuities which are both reasonable according to published information and if used in further analysis will result in predictions of reasonable accuracy.

A collection of case history studies categorized by geologic conditions would provide a means of selecting appropriate properties to predict the performance of excavations. Furthermore, in a continuing excavation (e.g. a subway) information obtained during its early stages can be used to calibrate the program in terms of appropriate properties to predict future performance. The results indicate that the program developed can be an extremely useful aid in designing excavations in rock.

DOD IMPLICATIONS

The evaluation of the structural stability of underground openings, ground support structures, and other facilities is an essential step both in the design and in the survivability/vulnerability assessment of underground structures and weapon systems.

A computer program has been developed to analyze the influence of excavation techniques, construction, and excavation sequence and structural support schemes on the stability of excavations in rock masses where the rock mass behavior is dominated by block slippage along discrete joint planes, or a global inability of the rock mass to resist tensile stress, or elastic-plastic behavior of the rock mass, or any combination of the three rock mass physical characteristics. This computer program should be considered as a tool to assist in the evaluation of supported and unsupported underground openings.

CONSIDERATIONS FOR FURTHER RESEARCH

Studies of a limited number of the case histories have indicated that the analytical models developed under this contract could predict the behavior of underground openings with reasonable accuracy. As indicated, the major problem in utilizing the computer code is the lack of information on the properties of geologic discontinuities.

By studying additional well documented case histories and using an iterative procedure appropriate material properties for different geologic conditions can be determined. Such information would be extremely valuable for future work and should be developed.

ACKNOWLEDGMENTS

The research was supported by the Advanced Research Projects Agency of the Department of Defense and was monitored by the Bureau of Mines. Dr. R. D. Singh and Mr. A. M. Abdullah contributed significantly to the research reported herein. The Project Officer of the Bureau of Mines was Mr. Grant Anderson; his assistance and cooperation in conducting this study is acknowledged.

The cooperation of Dr. Don Deere, Consultant to the project; Dr. Edward Cording, and Mr. J. W. Mahar of the University of Illinois, Urbana, in furnishing the writers with their original data on a rock tunnel of Washington, D.C. METRO analyzed in this study is gratefully appreciated. Assistance provided by Mr. Eugene H. Skinner of the Bureau of Mines in search of case histories of underground openings is also gratefully acknowledged.

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INTRODUCTION

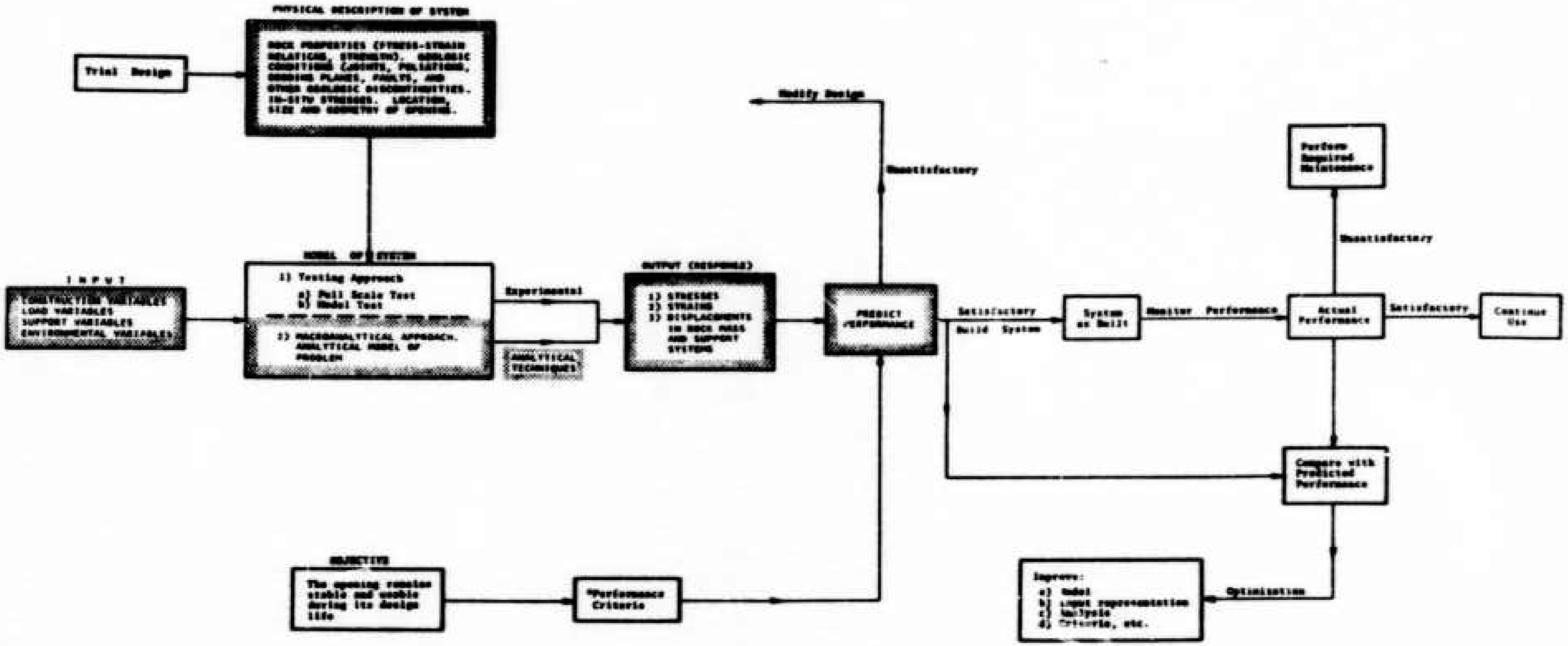
In the previous contract (HO210046), a general plane strain finite element computer program was developed for the analysis of underground openings in rock. Under the present contract, it is proposed to add to the capabilities of this computer program by including techniques to analyze the influence of excavation techniques, construction and excavation sequence, and structural support schemes on the stability of excavations in rock.

Because of the development of increased analytical capabilities, it is considered appropriate to consider these capabilities in the context of the total problem of evaluating the stability of openings in rock. Whereas the results of analytical studies can be of great assistance in the evaluation of stability, there are several other factors that enter into the evaluation, and it is necessary that one not be lulled into a sense of false security because of elaborate computational techniques.

METHODOLOGY FOR EVALUATING THE STABILITY OF OPENINGS IN ROCK

Analysis of openings in rock is a complex problem because of the numerous factors influencing the behavior and stability of the opening. These include (i) rock properties, (ii) the location, geometry and size of the opening, (iii) geologic conditions such as joints, foliation surfaces, bedding planes, shear zones and fault zones, (iv) in-situ stress conditions, (v) excavation and construction methods, and (vi) support systems. Because of the complexity of the problem, it is appropriate to develop a general framework for evaluating the stability of openings in rock. Such an approach is summarized in Figure 1. This approach consists of the following major steps:

1. Establishment of Objectives and Performance Criteria. In any design process, it is necessary to establish the objectives of the design and to translate these objectives into performance criteria. In general, the objective of the design



*Performance criteria are based on the displacements and strains which the rock mass and supports surrounding the opening can withstand safely.

FIGURE 1 - A GENERAL APPROACH FOR ANALYSIS OF UNDERGROUND OPENINGS

is that the opening remains stable and usable during its design life. The performance criteria, for example, may be in terms of limiting the stress, strain and/or displacement induced in the rock mass and support system surrounding the opening.

2. Definition of Input and Output (Response) Variables. The major inputs can be considered in terms of loads, external and internal, e.g., in-situ stress, the effects of construction methods and excavation techniques including environmental (temperature and moisture) factors. The output variables may be stress, strain and displacement in the rock mass and support system. In defining the output variables, it should be recognized that they should be in terms of the performance criteria in order that a comparison can be made.

3. Physical Description of the System. The description of the system consists of the following: (a) the size and geometry of the opening, (b) its location below the ground surface, (c) distribution of geologic discontinuities (e.g., joints, foliation, bedding planes, faults, shear zones) of the rock mass, (d) mechanical properties of the rock mass, and (e) support systems used to maintain stability.

4. Determination of the Response of the System. This requires (i) the development of a model or idealization for the system, and (ii) the use of analytical and experimental techniques to determine the response of the model to the prescribed inputs. There are two general approaches for determining the response of the system.

(a) Experimental Approach - Laboratory models using photoelastic techniques or blocks of rock-like materials tested under simulated field conditions or a full scale test conducted in the field may provide data which can be useful in understanding the behavior of the real structure and developing an empirical design procedure for openings constructed under similar geologic conditions. However,

it is often difficult to extrapolate an empirical design procedure to conditions different from those under which the procedure was developed.

(b) Macroanalytical Approach - This approach involves the development of a mathematical model for the system and the solution of an appropriate boundary value problem. The development of such a model should be based on a physical understanding of the problem and an evaluation of past performance. The objective of such an approach is to develop a general, theoretically sound method of analysis on the basis of which the output of the system can be determined if the input is prescribed and the system adequately described. It is the macroanalytical approach that forms the basis for the majority of the existing design methods in engineering practice. *It is within the context of this approach that the analytical techniques under development in this contract have to be viewed.*

5. Decision on Acceptability of Design. The predicted output of the system should be compared with performance criteria to see if stress, strain and displacement in the surrounding rock mass and support system are within allowable limits to prevent failure of the support system and the opening.

6. The Feed Back Loop - Optimization. In the idealization of a physical system as complex as an underground excavation in a rock mass, it is necessary to make many simplifying assumptions. In order to establish the validity of these assumptions, it is necessary to compare the performance of the actual system with the predictions. This comparison is essential to establishing the reliability of the techniques developed for evaluating stability. The results of monitoring the performance of the actual system should then be fed back into the methodology for evaluating stability to improve the assumptions and idealizations. In this manner the methods for evaluating stability

will improve. In this context the development of a theoretically sound method for evaluating the stability of openings in rock is an iterative process as indicated in Figure 2.

The work performed under this and the previous contract are indicated in the context of the total system in Figures 1 and 2. As more information is obtained from observed performance and better analytical techniques are developed, there will undoubtedly be further improvements in the methods of evaluating stability.

OBJECTIVES

The objectives of this study are:

(1) To incorporate into the finite element computer program for plane strain analysis, with capabilities to perform joint perturbation, no tension and elasto-plastic analyses developed under Contract No. H0210046, the capability to model and analyze:

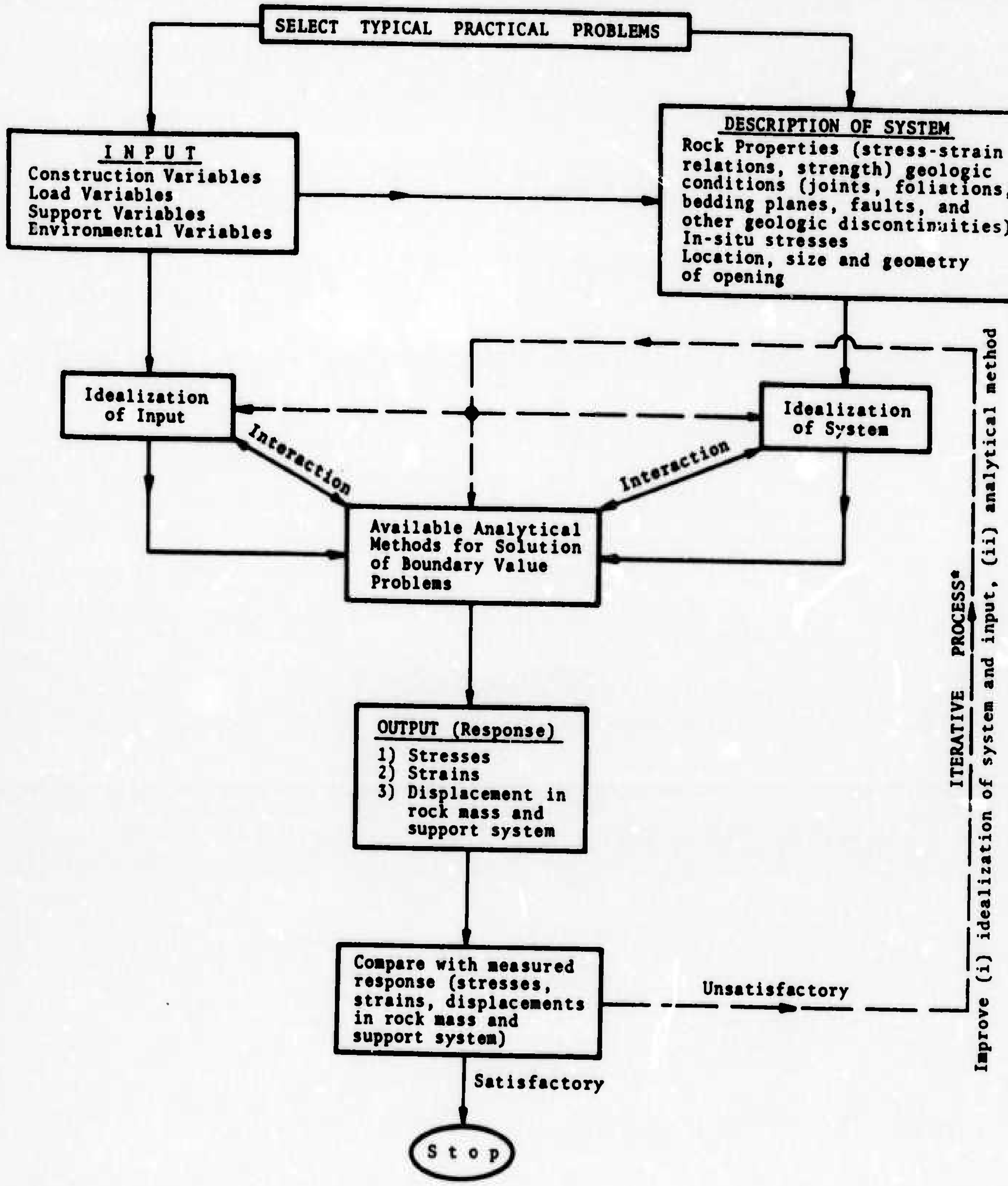
(a) structural support schemes used in the construction and design of tunnels, and

(b) excavation techniques and construction and excavation sequence in underground construction;

and

(2) To evaluate the analytical method developed in (1) by a study of case histories.

It is not proposed to make any basic modifications in the previously developed program but rather to add to its capabilities. Therefore, the additional capabilities have to be incorporated into the program within the context of the limitations of the previous program; the major limitations being (i) plane strain



*1. As a first iteration linear elastic analyses were utilized.

2. Comparison with observed performance indicated that the linear elastic analysis was not satisfactory.

3. We are now in the second iteration having improved our analytical methods.

FIGURE 2 - DEVELOPMENT AND VERIFICATION OF ANALYTICAL METHODS FOR ANALYSIS OF UNDERGROUND OPENINGS

conditions have to be assumed, and (ii) time dependent material properties cannot be included. The emphasis of the computational models is to the design and construction of tunnels.

RESEARCH APPROACH

In order to meet the objective of the contract, the following general research approach was adopted:

(i) Review, *for the purpose of developing computational models*, the available information on excavation techniques, construction and excavation sequences, the mechanisms of ground support and the current design techniques for support systems.

(ii) Formulate the computational models on the basis of the review in (i).

(iii) Modify the computer program to incorporate the computational models.

(iv) Select case histories for analysis.

(v) Idealize the case history problems for analysis and conduct analysis.

(vi) Compare actual performance with predicted performance.

(vii) Formulate conclusions on the adequacy of the computational method based on (vi) and recommend improvements.

The report is organized in accordance with this approach.

EXCAVATION TECHNIQUES, CONSTRUCTION AND EXCAVATION SEQUENCE AND SUPPORT SYSTEMS

In order to develop models to include excavation methods, construction sequence and support systems, it is first necessary to review existing information in these areas so that the essential elements that should be modelled can be identified.

EXCAVATION TECHNIQUES

The drill and blast method and the boring machine are two methods commonly used in rock excavation. Excavation by blasting causes loosening and fracturing of the rock beyond the excavated boundary. The depth of the disturbance depends on the blasting technique and rock conditions, and may be estimated by empirical methods based on experience or by seismic refraction surveys in the field. Seismic refraction surveys along the tunnel walls (Deere, et al. 1969) have shown a 2- to 10-ft-thick low velocity zone which is considered to be disturbed by blasting. The thickness of this zone is a function of the rock quality. The loosening and the thickness of the zone of disturbed low velocity rock increases as the rock quality decreases.

Compared to the fracturing and loosening of the rock by blasting, the boring machine causes little or no disturbance. The rock immediately adjacent to the opening can be assumed to have essentially the same properties as that of the undisturbed rock.

It appears, therefore, that the essential capability that should be developed in modelling the excavation technique is the ability to model loose and fractured rock in the vicinity of the opening.

CONSTRUCTION AND EXCAVATION SEQUENCE

The construction sequence is the sequence in which the excavation is conducted and the support system installed. The effect

of the time scale of these operations depends to a large degree on the time-dependent response of the rock.

Tunneling causes changes in stress and gradual loosening in the vicinity of the opening. The gradual loosening depends not only on the quality, bedding, jointing and foliation of rock, as well as the width of the excavation, but also on the distance between the last support and the rock face. For a certain period prior to breakdown, the loosened rock itself is capable of overbridging the unsupported cavity. This is referred to as the bridge-action period (t_b). The sequence and method of excavation should be selected to enable the installation of the necessary new support before the bridge-action period has expired. The typical position of the bridge-action period in relation to the sequence of operations during the construction of a tunnel excavated by blasting in certain rocks is shown in Figure 3. Figures 4 through 6 show the relation between time, overbreak and rock load for various rock types and support systems. The degree of overbreak, depending on the length of the unsupported section in a horizontally stratified rock, is illustrated in Figure 4. Figure 5 shows the progressive loosening in the supported section with time and the effect of backpacking on the rock load in blocky and seamy rocks. The relation between time, overbreak and rock load is presented in Figure 6.

The following techniques are commonly used in excavation of underground openings:

Full Face Method - In a full face operation, the tunnel is blasted out full size at each round. Small size tunnels always are driven full face.

Heading and Bench Method - In this method, illustrated in Figure 7, a top heading is carried ahead of the bench about 1-1/2 times the length of one round, usually about 6 to 16 feet. The heading

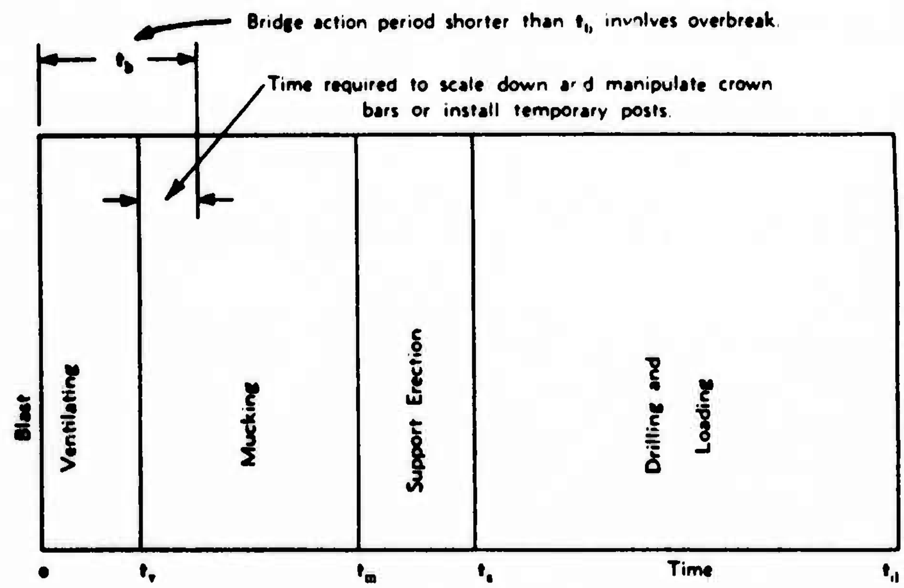


FIGURE 3 - DIAGRAM REPRESENTING OPERATING CYCLE FOR ONE ROUND (AFTER TERZAGHI, 1946)

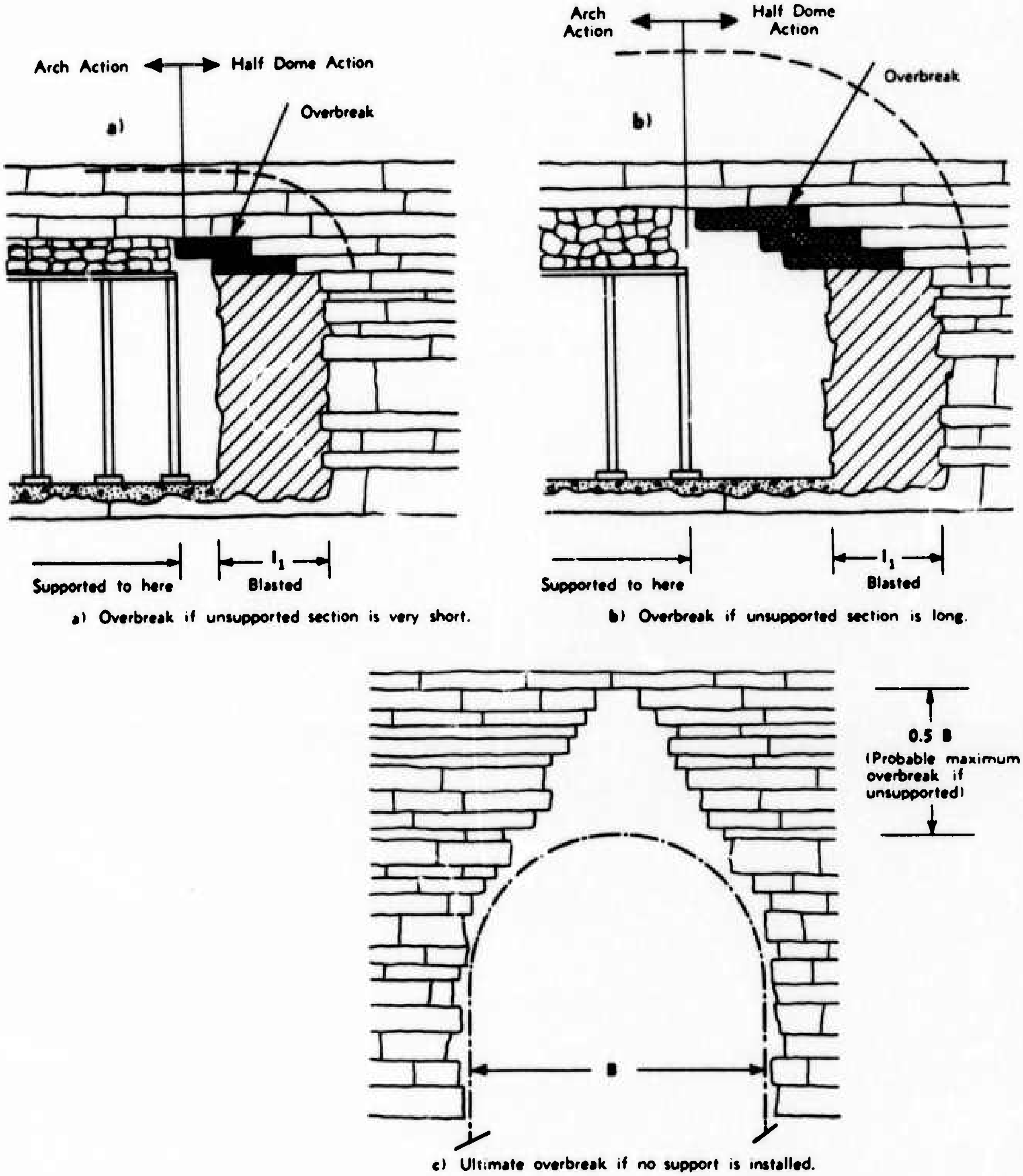


FIGURE 4 - OVERBREAK IN HORIZONTALLY STRATIFIED ROCK
(AFTER TERZAGHI, 1946)

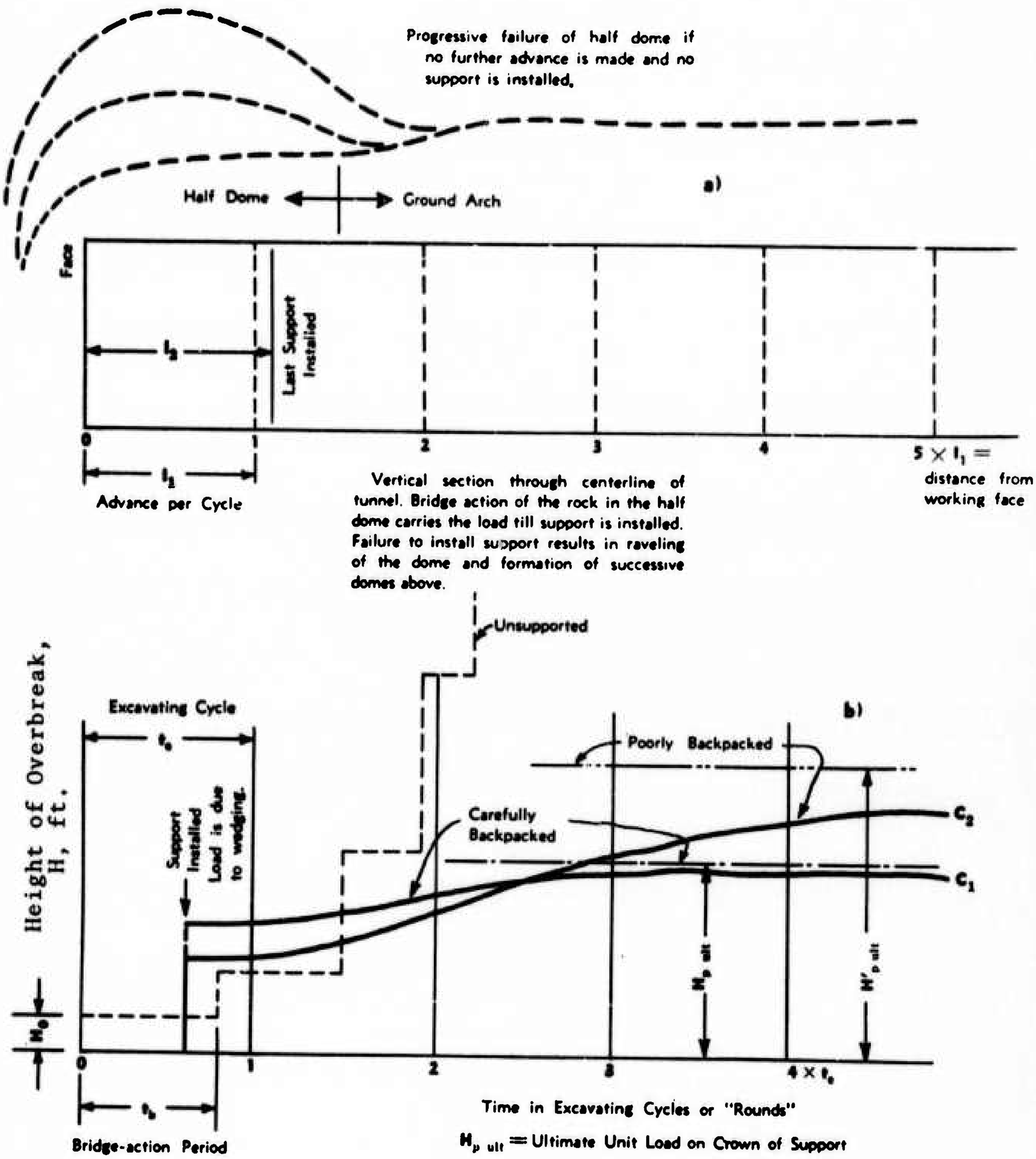


FIGURE 5 - RELATION BETWEEN TIME, OVERBREAK AND ROCK LOAD IN BLOCKY AND SEAMY ROCK (AFTER TERZAGHI, 1946)

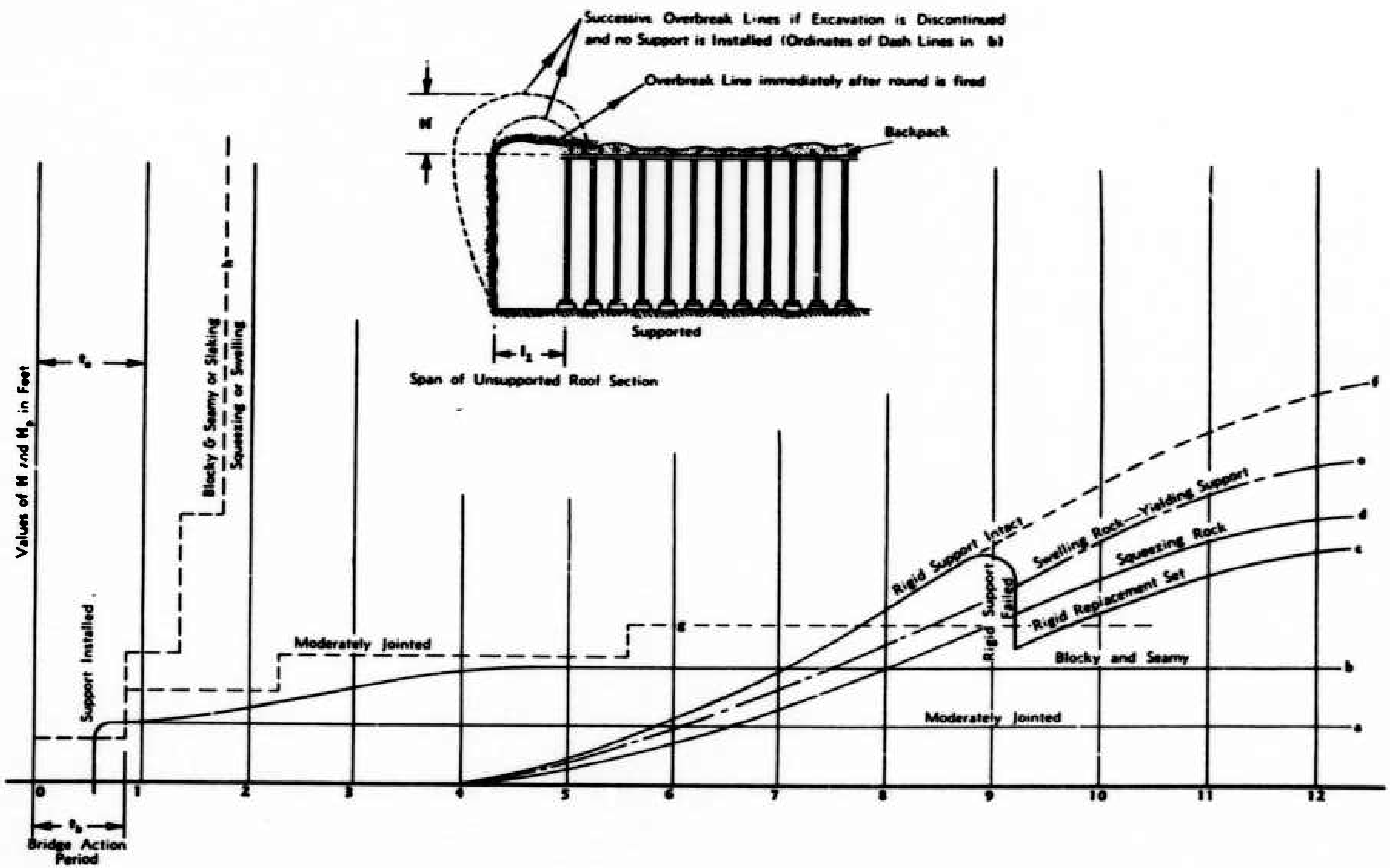


FIGURE 6 - RELATION BETWEEN TIME, OVERBREAK AND ROCK LOAD (AFTER TERZAGHI, 1946)

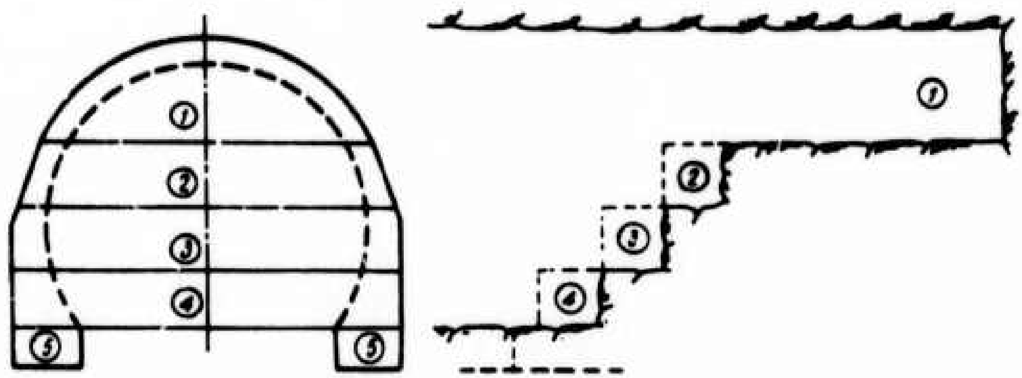


FIGURE 7 - HEADING AND BENCH METHOD (AFTER SZÉCHY, 1967)

has the full width of the tunnel and is carried down to the spring line. Both bench and heading are shot out at each round, the bench charges being fired first.

Top Heading Method - Instead of taking out the bench along with the heading, the top heading may be driven clear through as one operation, followed later by removal of the bench.

Side Drift Method - The side drift method, as shown in Figure 8, is sometimes employed in a large size tunnel through bad rock which requires support before mucking out.

Multiple Drift Method - This method is usually a combination of side drifts and top drift. It is employed to get through crushed rock in fault zones which may behave like soil. A typical case is shown in Figure 9.

Excavation of a large-size tunnel or underground powerplant sometimes may follow a complicated sequence. Some typical excavation sequences commonly used are illustrated in Figure 10.

In simulating the actual excavation sequence by plane strain conditions, considerable engineering judgment is required. Because of the discontinuities that exist in rock masses and their nonlinear behavior, the construction sequence can have a significant influence on the stresses and deformations in the rock mass.

SUPPORT SYSTEMS

In order to consider various support systems and their mechanisms, it is first appropriate to review the general concept of the function of a support system. When a support system is installed, the stability problem becomes complex and involves rock-support interaction. The stress redistribution and the rock-support interaction will depend on the flexibility of the support system. Deere, et al. (1969) have presented a schematic relationship shown in

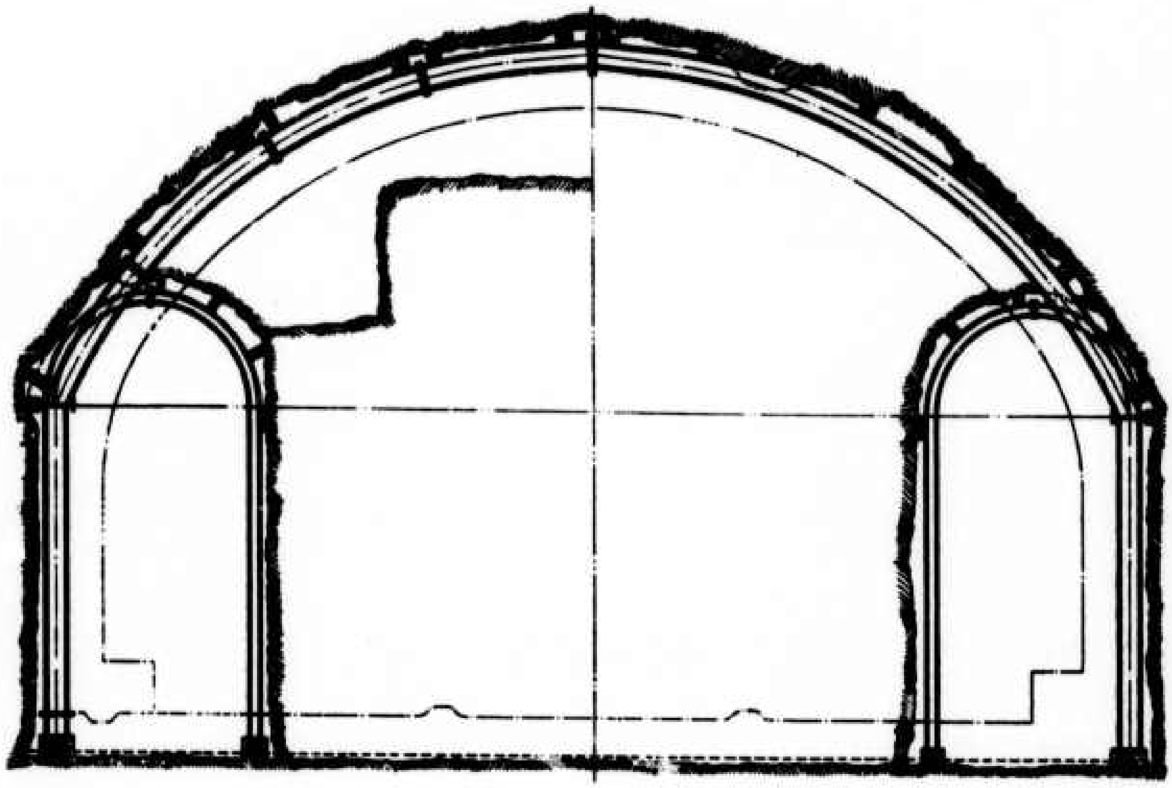


FIGURE 8 - SIDE DRIFT METHOD (AFTER PROCTOR AND WHITE, 1946)

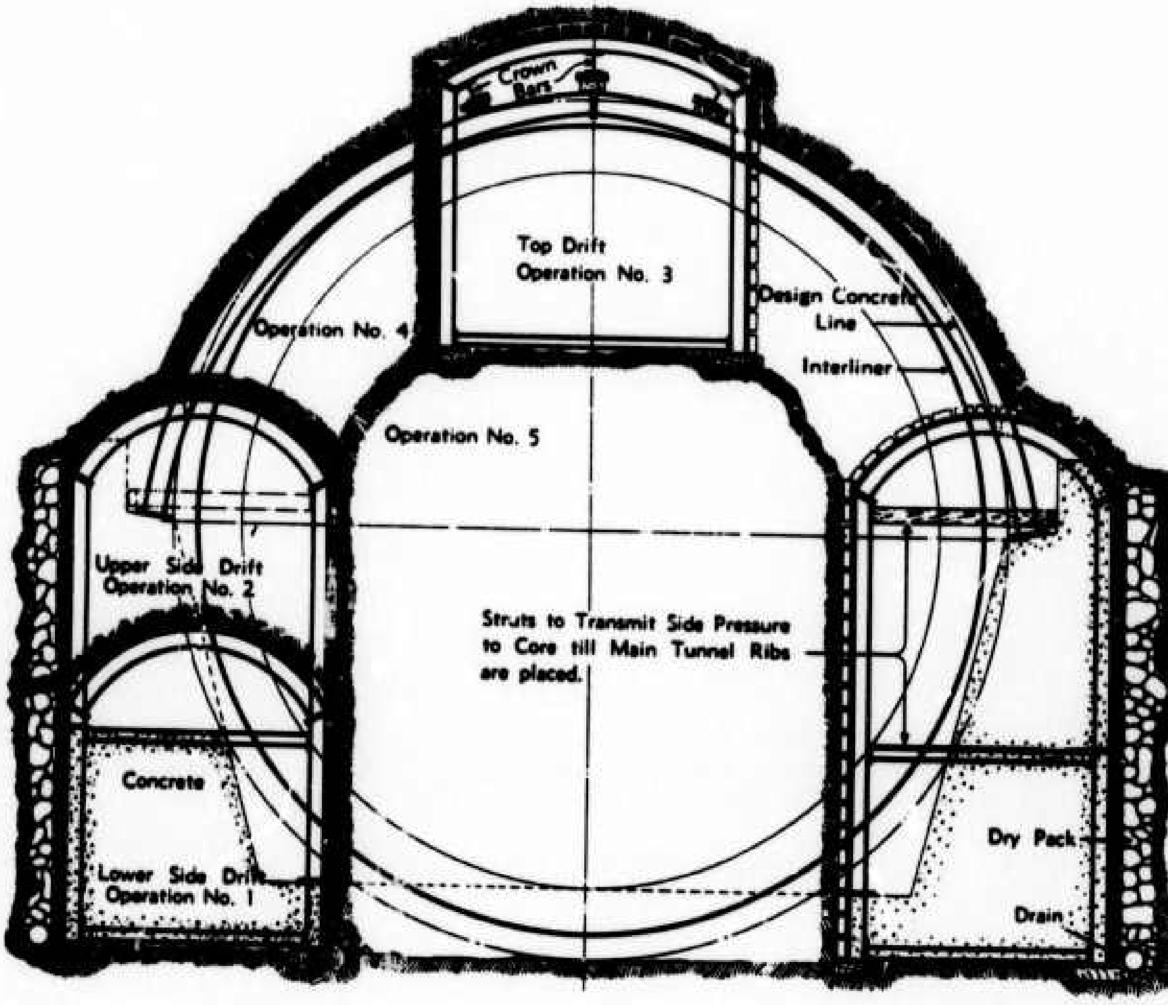


FIGURE 9 - MULTIPLE DRIFT METHOD (AFTER PROCTOR AND WHITE, 1946)

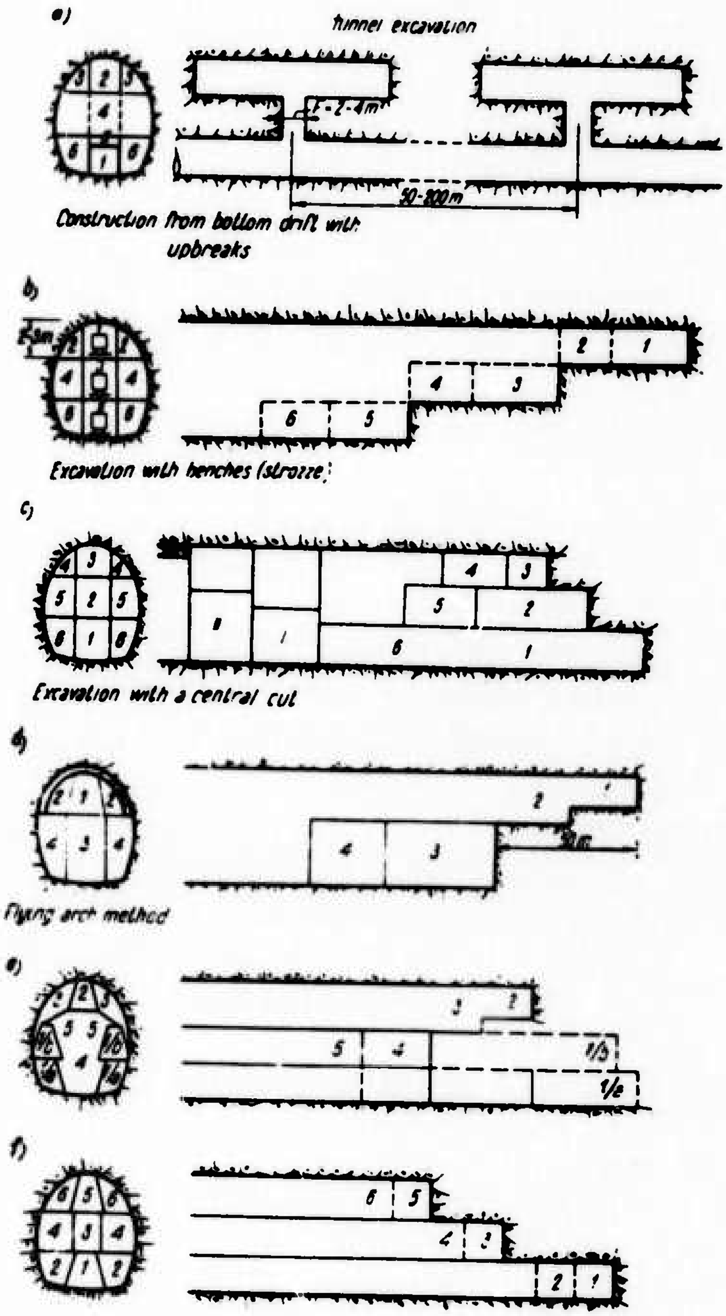
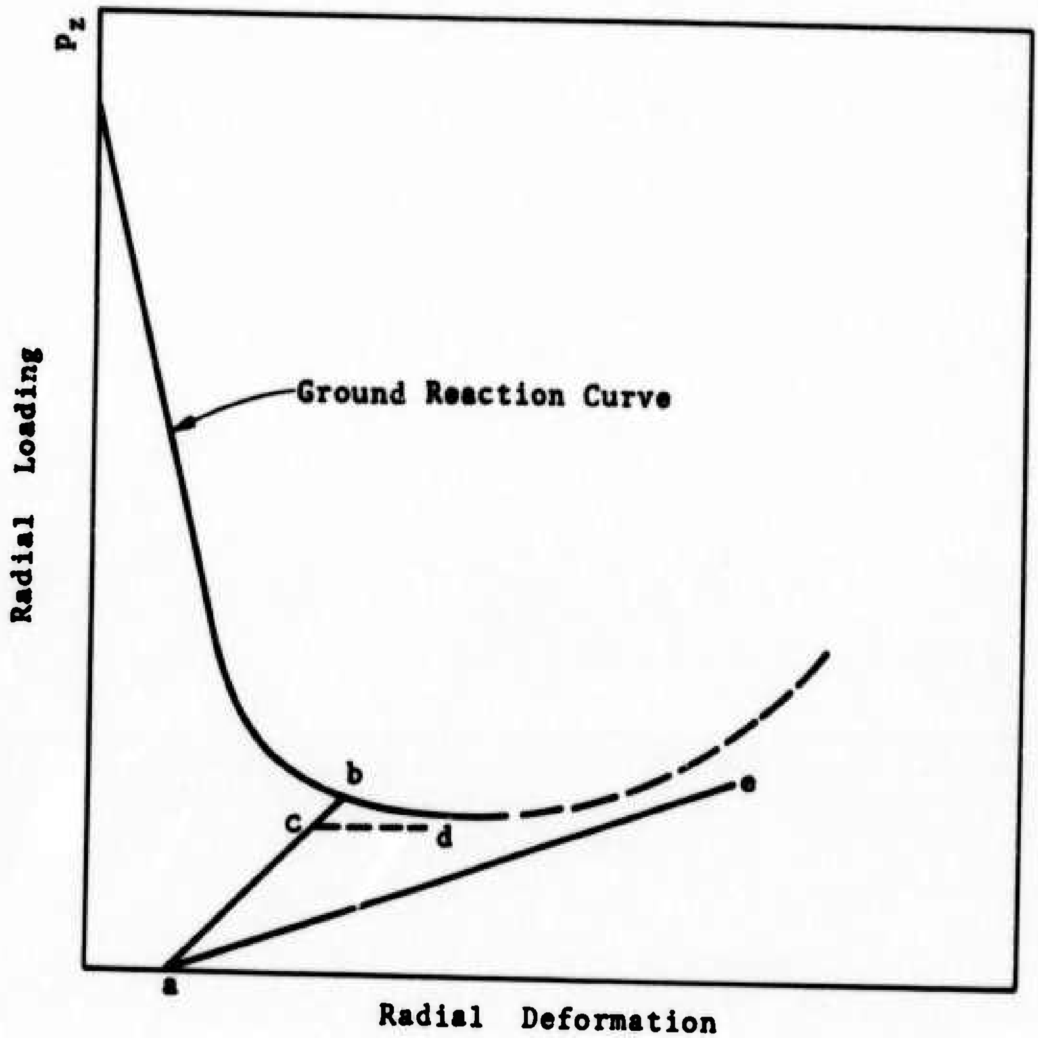


FIGURE 10 - TYPICAL EXCAVATION SEQUENCES

Figure 11 between the deformation in the rock, the time of installation, and stiffness of the support system. This relationship has been called the ground reaction curve. Certain basic concepts in the functioning of a support system can be explained on the basis of this curve. (i) If a support system is installed in time before the loosening of the rock occurs and it is so stiff that no yielding will occur in the support, the support system will be subjected to the initial stresses in the rock existing before the excavation is made. In reality, this case seldom occurs, because after the excavation and before a support system can be installed, the surrounding rock would have undergone some movements and redistribution of stresses. (ii) For a support installed at point a and with a stiffness represented by a-b, the opening will stabilize when the load on the support system and the radial deformation are represented by point b. (iii) A support with the same stiffness but insufficient load carrying capacity will yield and follow the path a-c-d without stabilizing the opening. (iv) A support that is too flexible will follow the path a-e without stabilizing the opening. (v) A support with a stiffness between a-b and a-e would stabilize the opening but might undergo an intolerable deformation. The true shape of the ground reaction curve is a function of the in-situ properties of the rock, the rock-support interaction, and the construction procedure.

Mechanisms of Ground Support

Excavation of an underground opening causes loosening of the rock and a redistribution of stresses in the vicinity of the opening. If the opening is unsupported and the rock in the immediate vicinity of the opening is in an unconfined state, it may be incapable of resisting the increased stresses. If this is the case, gradual loosening of the rock will occur leading to a consequent redistribution of stresses. The loosening zone shifts further inward until the magnitude of the increased stress decreases to a value lower than the strength of the rock. This process of the development of stress-relieved zone around the opening is schematically shown in Figure 12.



a-b Properly designed support
a-c-d Support yields before stabilizing opening
a-e Support to compressible

FIGURE 11 - GROUND REACTION CURVE FOR ROCK TUNNELS
(AFTER DEERE, ET AL. 1969)

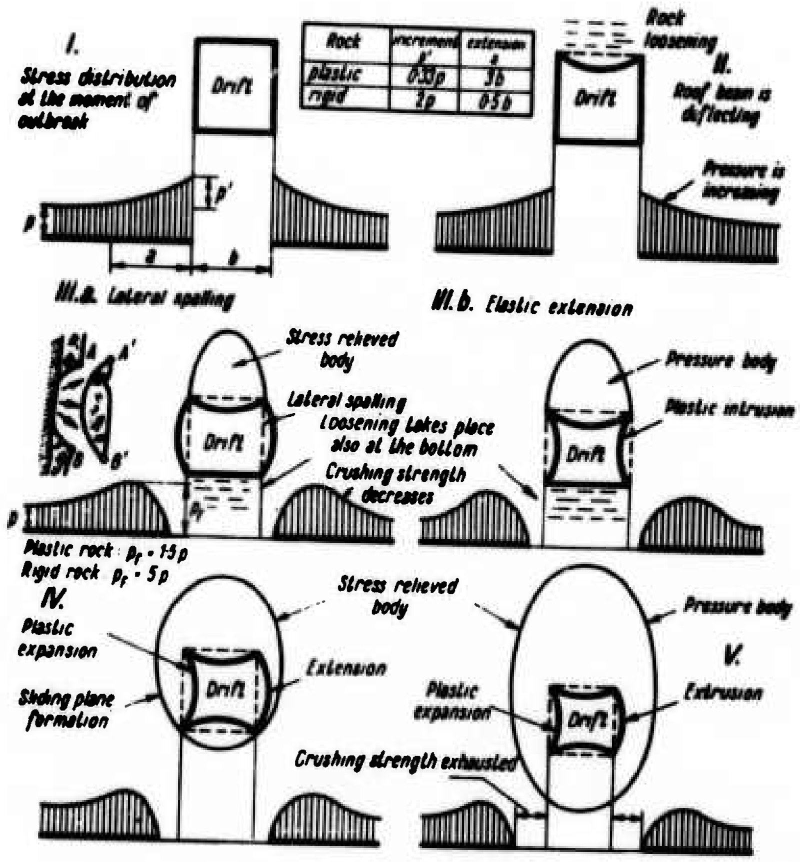


FIGURE 12 - DEVELOPMENT OF STRESS-RELIEVED ZONE OWING TO THE DEFORMATION PROCESSES OCCURRING AROUND THE CAVITY (AFTER SZÉCHY, 1967)

The purpose of the ground support is to supply the load-carrying capacity that the rock cannot provide and, thus, help the rock support itself. Because of interlocking and arching, a discontinuous rock mass in which an opening is excavated has a certain strength. Beyond a certain time limit (bridge-action period) and movement, the rock mass may become unstable. For a hard rock, the bridge-action period may be very short and the movement small when the maximum strength develops. The rock-support system is installed to provide sufficient load-carrying capacity before the bridge-action period expires or excessive movement has occurred. Because each type of support system requires different construction techniques and has different load-deformation characteristics, the ground-support interaction and, thus, the mechanisms of supporting the opening are different. The mechanisms of the ground support for steel sets, rock bolts, shotcrete and concrete liners are briefly described in the following paragraphs.

Steel Sets

Steel sets are commonly designed on the basis of Terzaghi's rock load concepts and are installed to support the weight of a rock mass that would fall out if unsupported.

A comprehensive discussion of the types, applications, design and construction of steel sets is contained in Proctor and White (1946). The following types of steel support systems have been developed and used for tunnels in rock, Figure 13.

(a) continuous rib types (leg and rib in one piece)
(b) rib and post type (arches on posts)
(c) rib and post wall type (arches on wall plates)
(d) rib wall plate and post type (arches on wall plates and posts)
(e) full-circle rib type

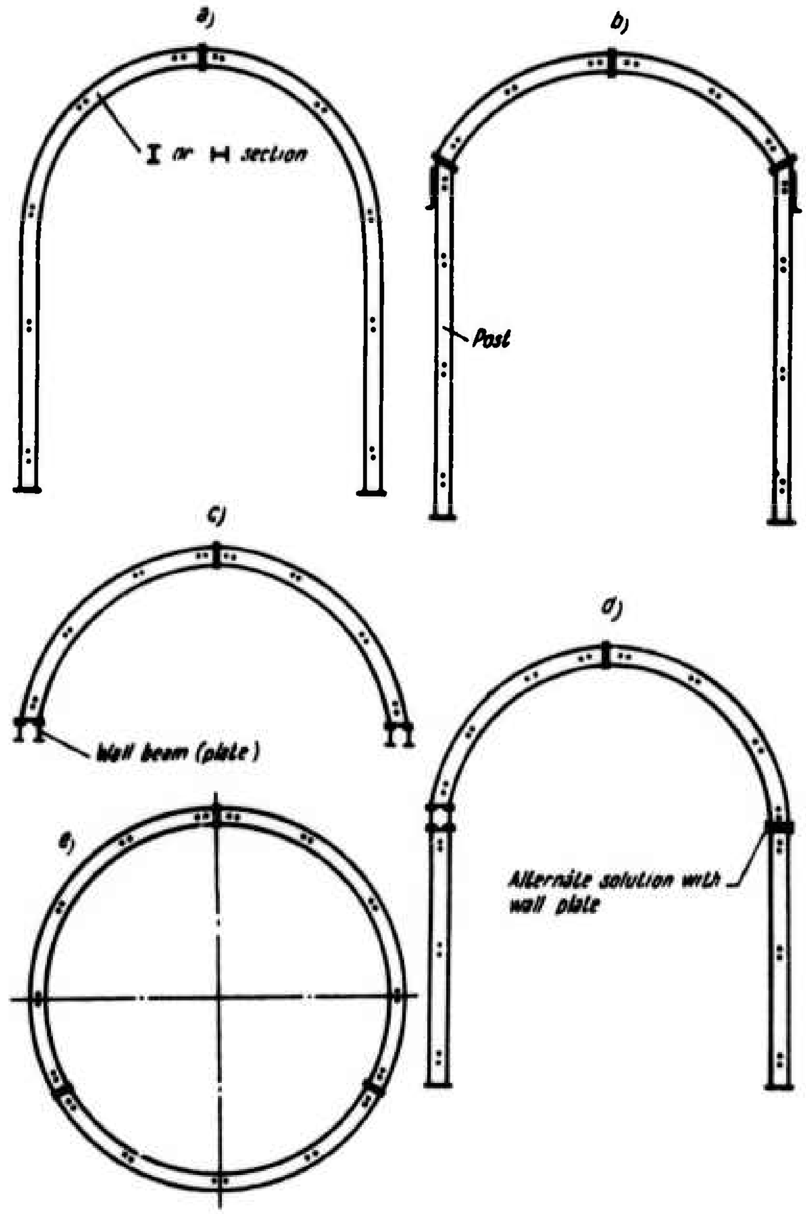


FIGURE 13 - STEEL PROPPING TYPES (AFTER PROCTOR AND WHITE, 1946)

Every one of the steel supports listed above consists of two or more different elements. These elements include the ribs, posts, wall plates, bracing and lagging, crown bars and truss panels. The functions of these elements are briefly described in the following paragraphs.

The rib, rib and post or rib, post and invert strut form a frame placed at right angles to the axis of the tunnel. The frames serve to receive the load and to transmit it to footings or to carry it by ring action as in full-circle ribs.

The wall plates serve as sills for the ribs. They transmit the load from the ribs through blocks or posts onto the rock. The lagging bridges the space between the ribs and is in direct contact with the rock. Thus, it transmits rock load to the ribs. The bracing is required to prevent buckling or shifting of ribs or posts.

The crown bars are located in the crown of the tunnel, parallel to the tunnel axis. After blasting and ventilating, they can rapidly be slipped forward, to support the newly exposed roof by cantilever action beyond the ribs. They may also be used to support the roof temporarily while the bench is being taken out.

The truss panels serve a function similar to the latter function of the crown bars. They are located at the spring line and constitute a temporary support for the ribs while taking out the bench, and are to be replaced by posts in the final stage of erection.

The steel sets are generally installed several feet behind the face of a tunnel and are spaced from two to eight feet on centers depending on the type of ribs and rock conditions. Because of construction techniques and structural flexibility, a steel support, in general, involves more rock loosening than rock bolting

and shotcreting. However, most supports are installed before the rock in the roof loosens all the way back to a stable arch i.e., before the bridge-action period expires. The load actually carried by the supports depends on the time the supports are installed and the amount of additional loosening that takes place after installation of the supports. This additional loosening depends on the type and quality of the support. Figure 14 shows that a typical support load varies with the rigidity of the support itself. Determination of support loads and, thus, the stability of the rock support system are very complicated because of the difficulty of modelling rock-support interaction.

Rock Bolts

Unlike steel sets, rock bolts and shotcrete are installed to help the rock support itself. Rock bolts can be installed at the working face directly after blasting and within a short time can exert a stabilizing pressure on the loosened rock surface. This early installation prevents the gradual relaxation or loosening of the decompression zone behind the new rock face.

The essential components of a rock bolt are the shank, the anchorage and the bearing plate assembly. Rock bolts are generally classified according to the type of anchor as sliding wedge and expansion shell. The process of rock bolting is to insert a shank in a hole drilled in rock and anchor the bolt in the bottom of the hole. The bolt is placed in tension between the anchor and the plate, thereby exerting a compressive force on the rock. The rock bolt is different from anchor bars which are grouted into holes in rock, but which are not prestressed.

The possible mechanisms of a rock bolt system in maintaining stability of an opening can be represented by the following two concepts.

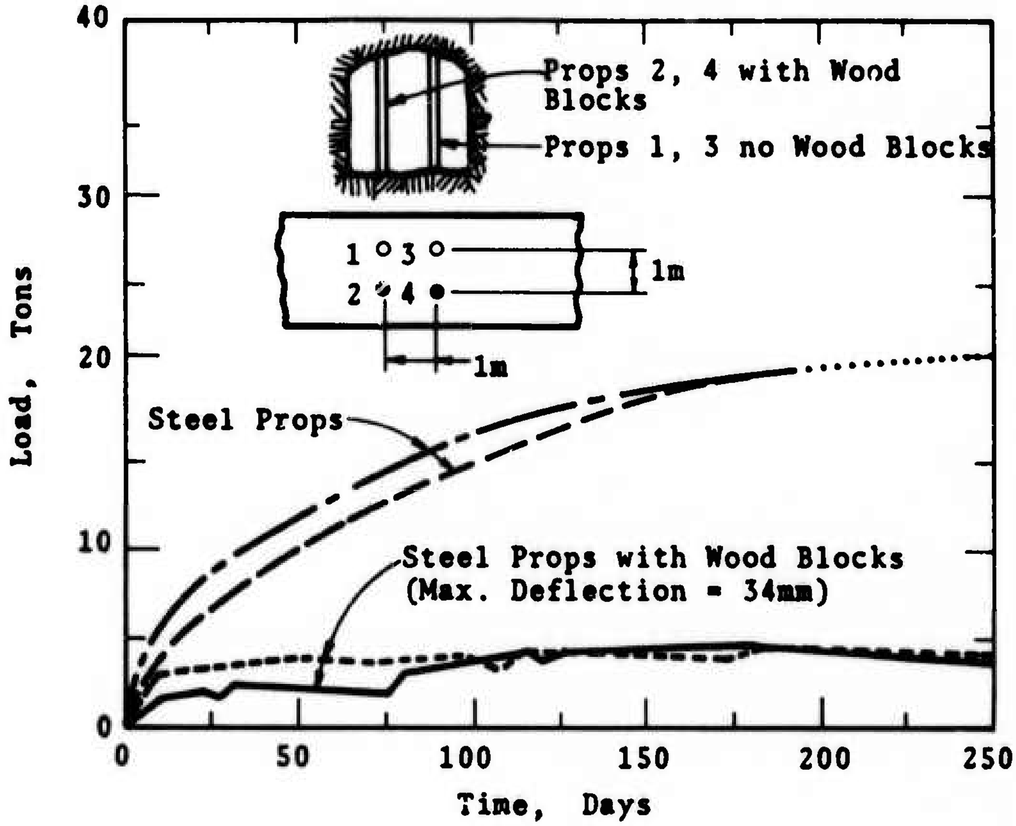


FIGURE 14 - EFFECT OF YIELDING SUPPORT ON SUPPORT LOAD (AFTER TAKAHASHI, 1966)

(a) Rock Support Concept - Rock bolts are used to secure loose joint blocks to solid ground to prevent the blocks from falling from the roof of the tunnel or spalling along the sidewall. Rock bolts may be placed in a horizontally stratified rock and spaced so that their combined strength is equal to the dead weight of the strata that would tend to fall. This concept of rock supporting may be called suspension. This type of support may be achieved by ungrouted, untensioned rock anchors or continuously grouted, untensioned reinforcing rods.

(b) Rock Reinforcement Concept - In this case, the purpose of the rock bolt is to confine the rock so that it will become a part of the total structure supporting the opening. This concept has been used to install the bolts in stratified rocks to bind the various strata together to act as a single beam capable of supporting itself and the overlying rock across the opening. In this case, the rock bolts are assumed to increase the friction and, thereby, prevent slippage between the beds, hence, forcing them to act as a beam. Extensive research has been done by Panek (1956a, 1956b, 1964) for design of bolting for a stratified roof. A typical bolting system is illustrated in Figure 15.

In the case of a fractured, jointed rock, when used in appropriate patterns, the bolts create a principal compressive stress normal to the free surface of the opening; and this, in turn, creates a zone of rock which acts as a structural membrane capable of providing its own support (Lang 1961). A schematic diagram illustrating the action of rock bolts on the rock around an excavation is shown in Figure 16. Figure 17 shows a typical pattern for rock bolts and its effect on the zone of the stressed membrane surrounding an opening.

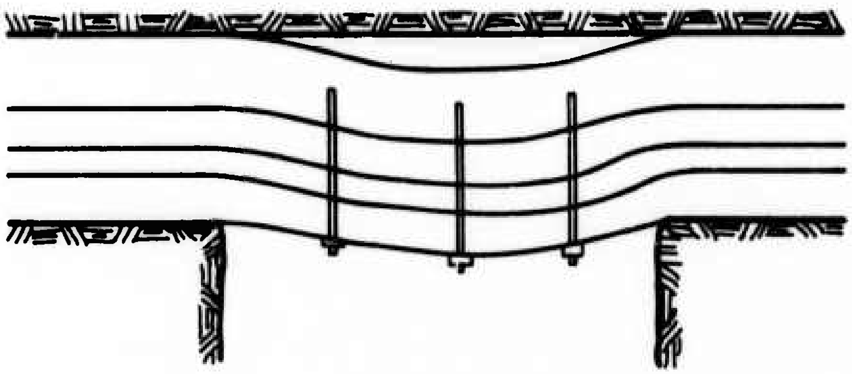


FIGURE 15 - TYPICAL ROCK BOLTING FOR A STRATIFIED ROOF

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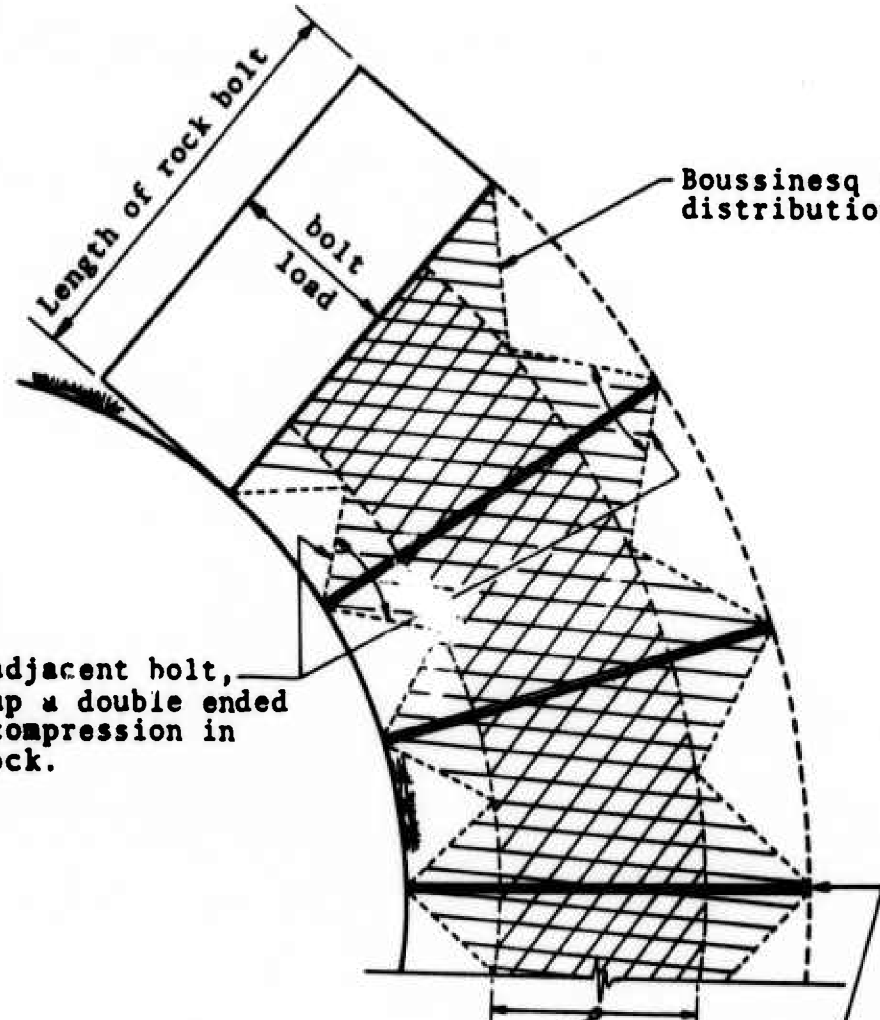


NOTES:

(1) Each adjacent bolt, sets up a double ended cone compression in the rock.

(2) The pressure cones overlap to form a "ring" of compressed and strengthened rock.

(3) The continued existence of the "ring" of tight rock is dependent on the continued existence of the load along the full length of the bolt.

FIGURE 16 - ACTION OF ROCK BOLTS ON THE ROCK AROUND AN EXCAVATION (AFTER PENDER ET AL. 1962)

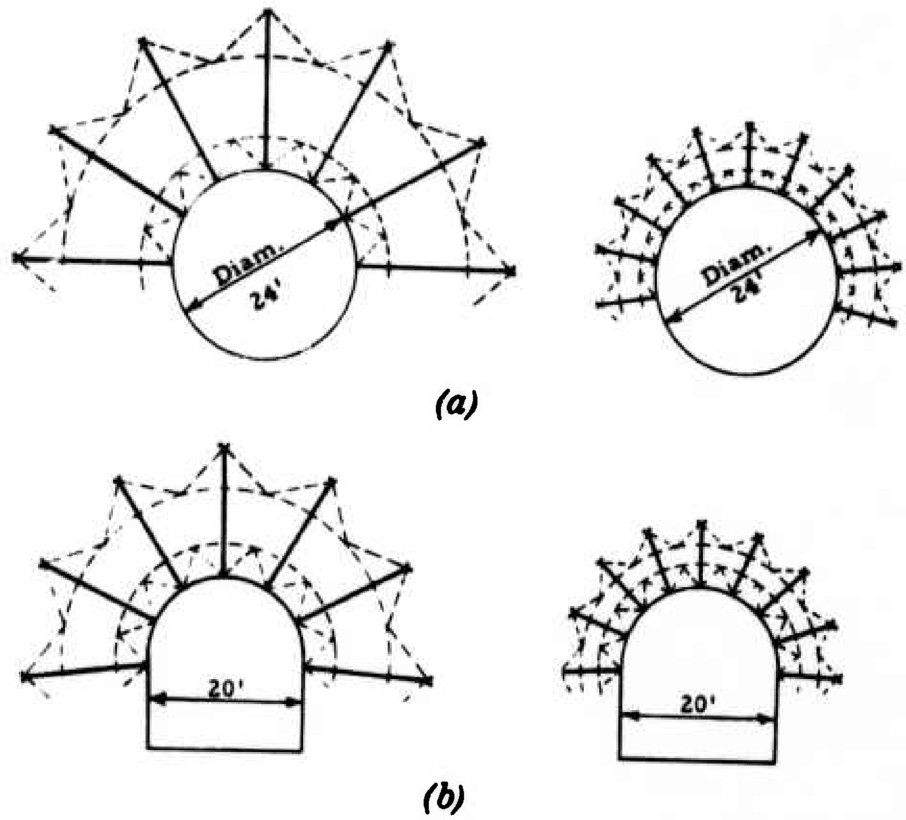


FIGURE 17 - TYPICAL PATTERN FOR ROCK BOLTS. A) TOP, SHOWS THE BOLT PATTERN FOR A CIRCULAR TUNNEL. THE SEVEN BOLTS SHOWN IN THE LEFT-HAND DRAWING WERE EACH 20 FT LONG, SPACED 6 X 6 FT. THE 11 BOLTS IN THE RIGHT-HAND DRAWING WERE EACH 8 FT LONG, SPACED 4 X 4 FT. B) BOTTOM, SHOWS ROCK BOLT PATTERN FOR A HORSESHOE TUNNEL. THE SEVEN BOLTS IN THE LEFT-HAND DRAWING WERE EACH 16 FT LONG, SPACED 5-1/2 X 5-1/2 FT. THE NINE BOLTS IN THE RIGHT-HAND DRAWING WERE EACH 8 FT LONG, SPACED 4 X 4 FT (AFTER LANG, 1961)

Shotcrete

Shotcrete may be defined as follows (Lorman 1968):

> Mortar or concrete that has been conveyed (by regulated air pressure or by positive displacement pump or screw) through a hose and discharged through a nozzle (usually hand held) at high velocity into a suitably prepared inflexible surface; the product, which has been premixed either dry (water added at the nozzle) or wet (water added prior to entry into the hose), is sufficiently stiff at impaction to support itself without sagging from an overhead surface or sloughing from a vertical surface.

Basically, fine shotcrete is mortar, and coarse aggregate shotcrete is concrete. Engineering properties of coarse aggregate shotcrete at age 28 days are similar to concrete (Lorman 1968).

The purpose of using shotcrete for ground support is to maintain the equilibrium and self-supporting capabilities of the rock surrounding the opening. Deere, et al. (1969) present a comprehensive discussion on the use of shotcrete for ground support. A layer of shotcrete is usually applied to a tunnel wall shortly after blasting. It provides continuous resistance to tunnel wall deformations. Several qualitative hypotheses for the mechanism of shotcreting support in a rock excavation have been suggested e.g., Alberta (1963, 1965):

(a) Shotcrete is forced into open joints, fissures, and seams and, in this way, serves the same binding function as mortar in a stone wall.

(b) Shotcrete hinders water seepage from joints and seams in the rock and, thereby, prevents piping of joint filling materials and air and water deterioration of the rock.

(c) Shotcrete's adhesion to the rock surface and its own shear strength provide a considerable resistance to the fall of loose rock blocks from the roof of a tunnel.

(d) A thicker shotcrete layer (15 to 25 centimeters) provides structural support, either as a closed ring or a fixed arch-type member.

The loads on a shotcrete liner are a function of the type and condition of the rock, the time of installation, rigidity of the support, and the interaction between the rock and the support.

Concrete Lining

Precast-concrete segments are commonly used for the support tunnels in soft ground. Except in a pressure tunnel, a concrete lining is used as a second or permanent liner of a tunnel in hard rock either to protect the first or temporary liner e.g., steel sets, rock bolts or shotcrete or to meet a secondary requirement such as improving the aesthetics, the acoustics or the aerodynamic flow properties of the tunnel. In this case, only the first liner is designed to support the total expected load. If a concrete lining is expected to carry some rock loads, its idealization is similar to that of shotcrete linings.

Current Design Techniques for Ground Support Systems

The purpose of reviewing the current design techniques is to establish how ground support systems are currently being modelled. Szechy (1967) and Deere, et al. (1969) discuss current design techniques for ground support systems. The current design techniques may be summarized as follows:

(1) Analysis of Unlined Openings - In this approach, the rock is considered as a continuum and the stresses and deformation around an unlined opening are investigated on the basis of an elastic or elastic-plastic analysis. Based on these analyses, the rock pressure on the support system is estimated.

(2) Rock Load Concept - The rock mass that is considered likely to fall if the opening is unsupported is determined. The weight of this rock mass is assumed to be the load that is to be carried by the support system. Basically, this is an empirical approach and does not consider rock support interaction. A typical example of this is Terzaghi's rock load theory.

(3) Support-Rock Interaction - Methods of analysis to account for effects of support-rock interaction on tunnel linings have been reported by Szechy (1967). Recently, Dixon (1971) has presented a similar technique to consider support-rock interaction in the analysis of tunnel support systems. This approach idealizes the rock mass with the Winkler-type foundation. Because of the limitations of the Winkler-type foundation in modelling the behavior of rock masses, this model is not considered to be realistic.

Current methods of tunnel lining and other support systems design tend to consider the estimate of rock load and the design of the structural lining as an independent process. The current design methods for steel sets, rock bolts and shotcrete lining are briefly described in the following paragraphs.

Steel Sets

Selection of the steel set support system depends on the following factors: (1) method of excavation, (2) rock behavior, and (3) size and shape of the tunnel cross-section. Design of steel sets is generally based on Terzaghi's rock load theory. Terzaghi (1946) defines rock load as the height of the mass of rock which tends to drop out of the roof of a tunnel. The magnitude of the rock load depends on the rock quality. Based on his experience on wood-blocked steel sets in tunnels excavated by conventional drilling and blasting techniques, Terzaghi (1946) established certain recommended design load ranges on the lining structure depending on certain rock classes. These are summarized in Table 1.

TABLE I

ROCK LOAD H_p IN FEET OF ROCK ON ROOF OF SUPPORT IN TUNNEL

WITH WIDTH B (FT) AND HEIGHT H_t (FT) AT DEPTH OF MORE THAN 1.5 $(B + H_t)$

(AFTER TERZAGHI, 1946)

Rock Condition	Rock Load H_p in feet	Remarks
1. Hard and intact	zero	Light lining, required only if spalling or popping occurs.
2. Hard stratified or schistose	0 to 0.5 B	Light support. Load may change erratically from point to point.
3. Massive, moderately jointed	0 to 0.25 B	
4. Moderately blocky and seamy	0.25 B to 0.35 $(B + H_t)$	No side pressure.
5. Very blocky and seamy	(0.35 to 1.10) $(B + H_t)$	Little or no side pressure.
6. Completely crushed but chemically intact	1.10 $(B + H_t)$	Considerable side pressure. Softening effect of seepage towards bottom of tunnel requires either continuous support for lower ends of ribs or circular ribs.
7. Squeezing rock, moderate depth	(1.10 to 2.10) $(B + H_t)$	Heavy side pressure, invert struts required. Circular ribs are recommended.
8. Squeezing rock, great depth	(2.10 to 4.50) $(B + H_t)$	
9. Swelling rock	Up to 250 ft irrespective of value of $(B + H_t)$	Circular ribs required. In extreme cases use yielding support.

Rock Bolts

A rock bolt support system is generally selected based on a consideration of the possible modes or mechanisms of failure of the rock around the opening. At present, there is no generally accepted design method.

Panek (1955, 1956a, 1956b, 1956c, 1962a, 1962b, 1964) has conducted an extensive study on design of bolting systems to reinforce laminated roofs. Figure 18 shows a design chart developed by Panek (1956b) for the reinforcement of a laminated horizontal roof on the basis of the development of friction between the layers resulting from the clamping action of tensioned rock bolts. The following factors are considered in the development of this design chart: (a) average bed thickness of mined roof, t_m; (b) length of bolts, h; (c) bolt tension and anchorage capacity, P; (d) number of bolts per set across the opening, N; (e) spacing of sets, b; (f) width of opening, L; (g) reinforcement factor, RF_t, or percent decrease in strata bending, $\frac{\Delta\varepsilon}{\varepsilon_{nfs}}$; and (h) coefficient of friction along planes of stratification, F.

Recently, McNiven and Ewoldsen (1969), Ewoldsen and McNiven (1969) and Goodman and Ewoldsen (1970) have attempted to analyze and design a rock bolt support system on a sound theoretical basis. This approach first computes the stress distribution in a rock mass surrounding an opening due to an installation of rock bolt reinforcement. In the stress computation, it is assumed that the rock mass is a linear elastic continuum. Stability along certain prevalent joint sets is examined by comparing shear strength and shear stress along the joint planes. Thus, an optimal design of the rock bolt system is achieved by an iterative process.

Shotcrete

At the present time, there is no rational design procedure for a shotcrete lining. Selection of the shotcrete support is largely based on experience and is a trial and error process. From the

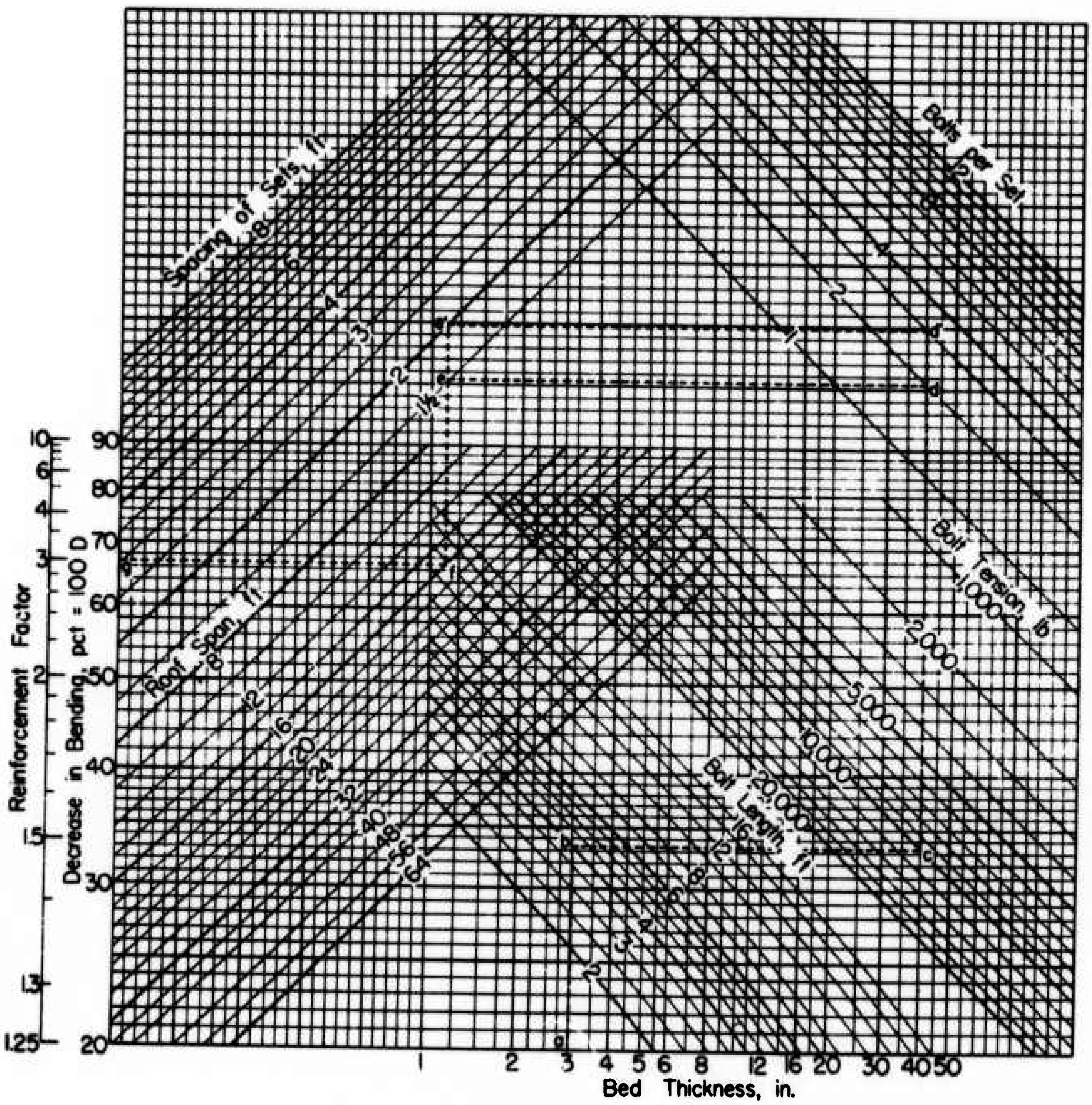


FIGURE 18 - ROOF-BOLTING DESIGN CHART (AFTER PANEK, 1956b)

accumulated experience with the use of shotcrete for underground support, some empirical design guides have been established for support selection. An example of these design guides is presented by Linden (1963), Figure 19. The design should be modified by local experience and geologic conditions. Deere et al. (1969) present a design approach based on structural consideration. This approach assumes that the liner is subjected to a uniform rock load. The desired lining thickness is adjusted to bring the combined thrust and bending stresses below the allowable values. Similar to the other rock support design, this approach does not directly consider the support-rock interaction.

SUMMARY - ESSENTIAL FEATURES TO BE MODELLED

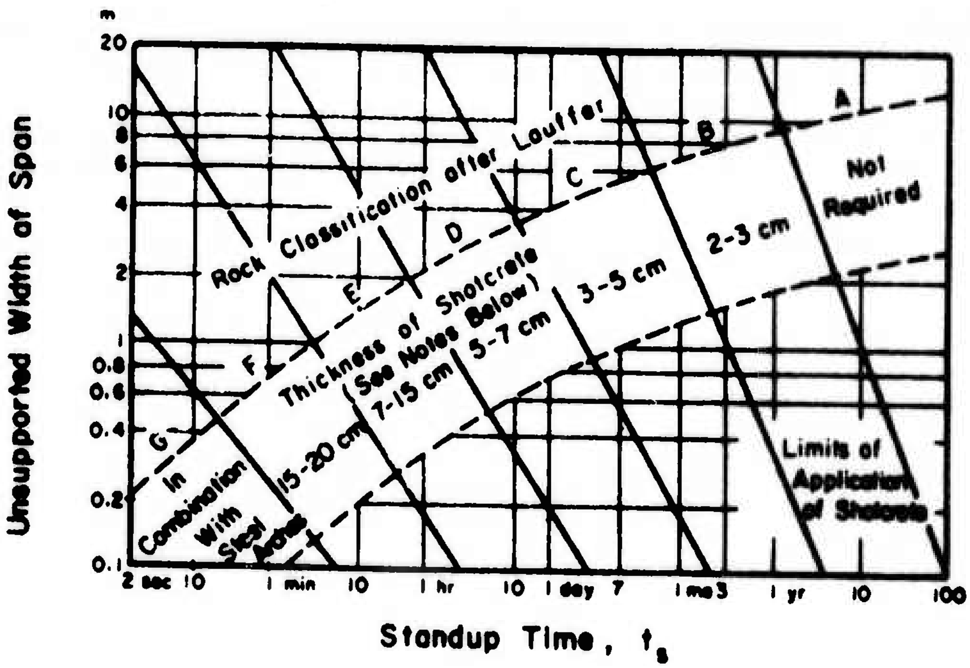
The purpose of this review was to establish the essential features in the excavation techniques, the construction and excavation sequence and the support system that have to be modelled. It is, therefore, appropriate that this summary identifies these essential features.

Excavation Techniques

Excavation of an opening creates some disturbance in a rock mass surrounding the opening. Depending upon the type of excavation methods, e.g., drilling and blasting, smooth wall blasting or boring machine, rock conditions and time of installation of support systems, zones of loosening and fracturing and depths of overbreak around the opening will be different. Generally, the drilling and blasting method causes more disturbance than other types of excavation. It is, therefore, necessary to be able to model the loosened and fractured rock in the vicinity of the opening.

Construction and Excavation Sequence

The time lag that occurs in installing any supports after the rock face has been exposed should be considered. The excavation sequence in which the excavation has many intermediate shapes



N O T E S:

(A) No support is required (Sound rock).

(B) Alternatively rock bolts on 1.5 - 2m spacing with wire net, occasionally reinforcement needed only in arch. (Sound, stratified or schistose rock, unstable after long time).

(C) Alternatively rock bolts 1 - 1.5m spacing with wire net, occasionally reinforcement needed only in arch. (Sound, stratified or schistose rock, unstable after short time).

(D) Shotcrete with wire net; alternatively rock bolts on 0.7 - 1m spacing with wire net and 3 cm shotcrete. (Strongly fissured rock, broken).

(E) Shotcrete with wire net; rock bolts on 0.5 - 1.2m spacing with 3 - 5 cm shotcrete sometimes suitable; alternatively steel arches with lagging. (Fully mechanically disturbed rock, very broken; gravel and sand).

(F) Shotcrete with wire net and steel arches; alternatively strutted steel arches with lagging and subsequent shotcrete. (Pseudo-sound rock, properties change with time; squeezing).

(G) Shotcrete and strutted steel arches with lagging. (Heavy squeezing, swelling rock, silt, clay).

FIGURE 19 - ROCK REINFORCEMENT WITH SHOTCRETE (AFTER LINDER, 1963)

before reaching the final shape should be considered. An intermediate shape may be more critical from a stability standpoint, and the stability of the final excavation may depend on the excavation sequence.

Structural Support Schemes

I. Geometry and Rigidity of a Support System

(a) Steel sets are generally installed as required by local rock conditions at specific spacings which may vary along the length of a tunnel. The spatial distribution and rigidity of steel sets along the length of the tunnel should be approximated in the computational model.

(b) A tensioned rock bolt exerts some localized three-dimensional effects on the stress-deformation of the rock mass in the vicinity of the rock bolt. Effects of bolt tension on the stress-deformation of the rock mass should be modelled. The presence of rock bolts and grouted rock bolts may increase the stiffness of the rock mass after support installation.

(c) Shotcrete or concrete linings are generally continuous along the tunnel axis. Their geometry and the structural support they provide must be modelled.

II. Support-Rock Connection (Interaction Effects)

(a) Steel Sets - Blockings which transfer loads between the rock and the support are placed at certain convenient discrete points on the steel set. The spatial distribution of blockings may affect the distribution of the bending and axial stresses in the steel set. It is necessary to model the blockings in the analysis.

(b) Rock Bolts - Bearing plates, anchorages and grouting along the rock bolt are the support-rock connections which may require consideration in the idealization of rock bolt systems. Goodman (1966) made a detailed study and concluded that effects of bearing plates are of little importance with respect to stresses more than two feet away from the free surface. This indicates that no appreciable error will result if the loading through bearing plates is approximated by point loads. This conclusion may also apply to anchorages. The grouting may affect the stiffness of the rock mass in the vicinity of the rock bolt.

(c) Shotcrete or Concrete Linings - Grouting or back packing behind a concrete lining should be modelled in analysis.

DEVELOPMENT OF COMPUTATIONAL MODELS

The computational models have to be formulated within the context of the finite element program developed under Contract No. HO210046. It is, therefore, appropriate to first describe briefly the existing program and its limitations in terms of modelling the excavation technique, the construction and excavation sequence, and the support systems.

BRIEF DESCRIPTION OF EXISTING FINITE ELEMENT PROGRAM

The existing finite element computer program which was developed under Contract No. HO210046 and which will be modified for the present project has capabilities to perform the following analyses under plane strain conditions in addition to a linear elastic analysis.

1. No Tension Analysis - The program is capable of performing a no tension analysis similar to that developed by Zienkiewicz, et al. (1968) and modified by Chang and Nair

(1972). A rock may be assumed to be capable of sustaining a limited amount or no tensile stress. This condition may occur due to the presence of numerous cracks and fissures.

2. Joint Perturbation Analysis - A one-dimensional joint formulation similar to that developed by Goodman, Taylor, and Brekke (1968) was used in the program. The joint is assumed incapable of resisting tensile normal stress and has a certain shear strength under a compressive normal stress. The shear strength of a joint is expressed by:

$$\tau_f = C + \sigma_N \tan \phi_e$$

where:

C = cohesion along the joint,
ϕ_e = effective friction angle of the joint (Patton 1966),
σ_n = compressive normal stress across the joint.

3. Elasto-Plastic Analysis - In the elasto-plastic analysis, the rock is assumed to be an elastic perfectly plastic material. The yield function utilized is a generalization of the Mohr-Coulomb hypothesis suggested by Drucker and Prager (1952) and is represented by the following equation:

$$f = \alpha I_1 + \sqrt{J_2} = k$$

where:

α and k = material constants,
I_1 = first stress invariant,
J_2 = second invariant of stress deviation.

The above-mentioned analyses may be performed concurrently depending on the idealization of the actual structure.

LIMITATIONS OF EXISTING PROGRAM

The limitations of the program in modelling excavation techniques, construction and excavation sequence and support systems can be divided into two broad categories: (i) geometry, and (ii) material properties.

Geometry

The existing program is limited to plane strain problems; therefore, any computational model has to develop idealizations which are compatible with the plane strain assumption. The following paragraphs discuss briefly how this assumption influences the development of computational models for excavation techniques, construction and excavation sequence and support systems.

Excavation Techniques

The modelling of the effects of excavation techniques is not influenced by the geometrical limitations except that the modelling is only valid at a sufficient distance from the actual excavation face. Therefore, the immediate effects of the utilization of a particular excavation technique cannot be modelled.

Construction and Excavation Sequence

The construction and excavation sequence refers to the shape of the openings at various times and the installation of the support systems at various times. Again, the major limitation is that the sequence near the face of the excavation cannot be modelled.

Support Systems

In addition to the limitation of modelling the system at a sufficient distance from the excavation face, there is the additional problem that support systems are in general not continuous. Rock supports using shotcrete or concrete lining may be modelled as continuous supports along the axis of the opening. However, support systems utilizing rock bolts or steel sets would have to be modelled on the basis of various idealizations to fit them within the framework of a plane strain analysis.

Material Properties

The existing program is limited to materials with time-independent properties. The influence of this limitation on the development of a computational model arises from the fact that the excavation, construction sequence and support system installation occurs over a finite time interval. Any time-dependent material response that would occur over this period cannot be directly accounted for.

GENERAL MODELLING CONCEPTS

Excavation Techniques

It has been discussed in the previous sections that excavation of an opening creates some disturbance in a rock mass surrounding the opening. Depending upon the excavation technique, e.g., drilling and blasting, smooth wall blasting or boring machine, rock conditions and time of installation of support systems, the zones of loosening and fracturing and depths of overbreak around the opening will be different. Zones of disturbance may be estimated on the basis of experience at locations with similar geologic conditions and excavation methods or determined by seismic refraction surveys in the field.

The essential features that have to be modelled in simulating the effects of excavation techniques are the following:

(i) The stress free excavation face.
Dunlop, Duncan and Seed (1968), Chang and Duncan (1970), Clough and Duncan (1969), and Chang and Nair (1972) have shown that excavation may be simulated in the finite element method by applying stresses to the boundary exposed by excavation so that there is no resultant stress on the excavation face. A similar technique will be used in this study.

(ii) The disturbed zone in the vicinity of the excavation. This zone can be modelled by assuming a lower modulus for the material in the zone or by assuming that the material is incapable of carrying any tensile stress. Both these techniques will be utilized in this study.

<u>Construction and Excavation Sequences</u>

The essential features to be modelled and the basic concepts in modelling them are described below:

(i) The time sequence of construction, including installation of supports.

Because of the limitation that only time-independent material properties can be included in the program, the time sequence of construction will be modelled in accordance with the following two stage analyses.

(a) An initial analysis prior to any support installation will be conducted.

(b) A subsequent analysis will be conducted with the support system installation treating the results of the analysis in (a) as the initial condition. In a practical problem, it will be necessary to bracket possible initial conditions.

(ii) The excavation sequence.

The opening goes through many shapes before reaching the final shape. If the problem could be treated as linear elastic, the final stress distribution would be independent of the excavation sequence; for non-linear problems, it is necessary to consider the sequence. Excavation sequence will be simulated by removing those elements that will be excavated and ensuring that the excavation face is stress free.

Support Systems

This development is based on considering the interaction of the support and the surrounding rock mass. The three basic support systems considered in this study, (i) steel sets, (ii) rock bolts, and (iii) concrete and shotcrete liners, are discussed separately.

(i) Steel Sets - A series of beam elements which are capable of carrying both bending and axial stresses may be used to idealize a steel set. The support-rock connections, i.e., blockings, may be idealized by a one-dimensional or a regular element if the connections are to transfer axial forces or both axial and shear forces. As described previously, this study is confined to analysis of plane problems; and, thus, both the opening and its support system are to be idealized as plane strain problems. It is proposed that the sets along some length of the tunnel be idealized by a continuous support with a section modulus equivalent to the average section modulus of the sets. The blockings are assumed to be continuous along the length of the tunnel. The idealization of the steel sets is illustrated in Figure 20.

(ii) Rock Bolts - Because of the difficulties associated with analysis of a rock bolt system, i.e., the three-dimensional aspect, the interaction of each rock bolt with the rock will not be modelled in this study. The following approximations are proposed to idealize the rock bolt support system:

(a) To increase the stiffness of the rock mass in the immediate vicinity of rock bolts to account for the presence of rock bolts and grouted rock bolts.

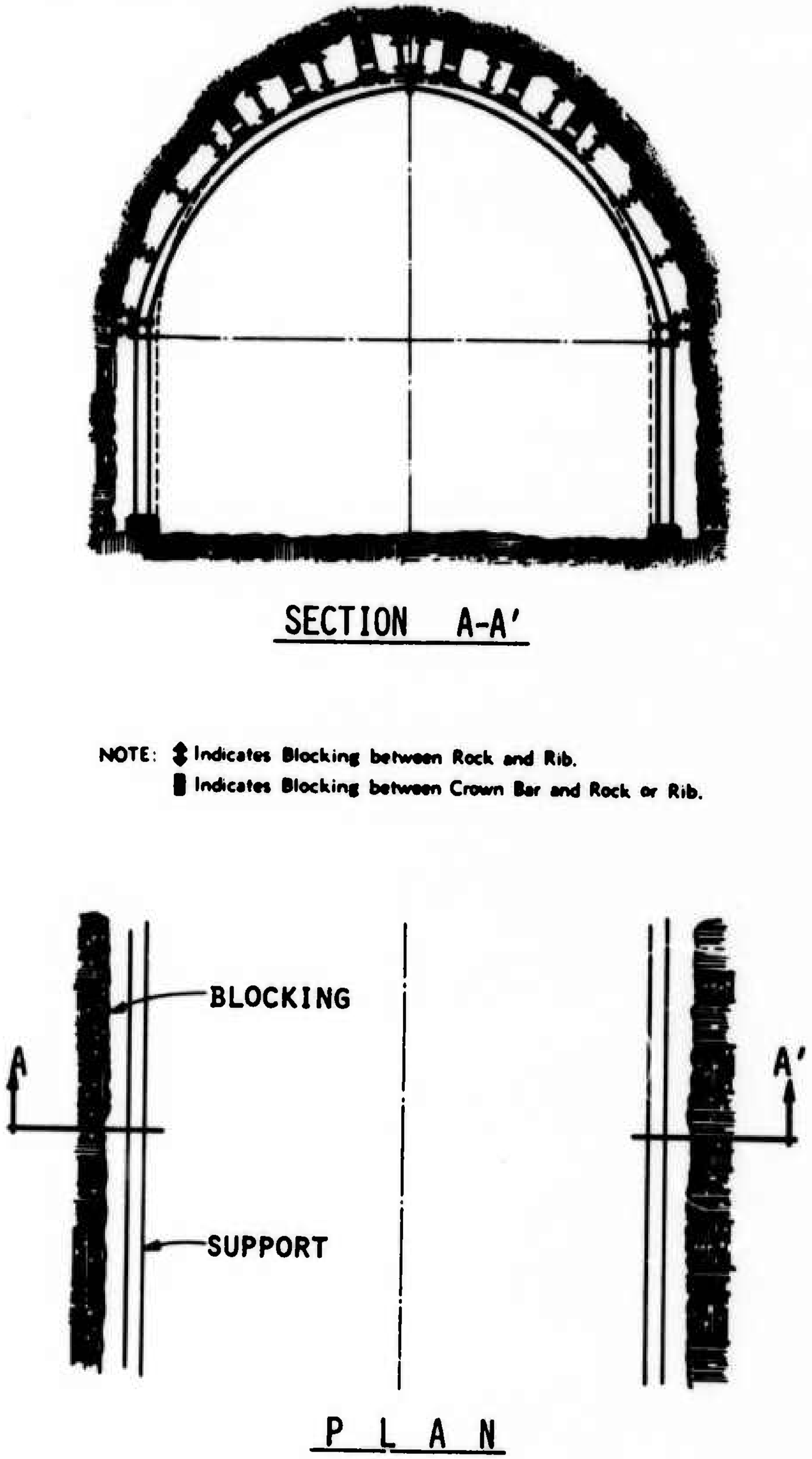


FIGURE 20 - IDEALIZATION OF STEEL SETS FOR PLANE ANALYSIS

(b) To approximate the effects of tensioned bolts on the rock mass by applying a set of opposite concentrated loads at the anchor and bearing plate. Each concentrated load is considered to be an equivalent line load along the tunnel axis to represent a row of rock bolts. The magnitude of the line load is determined by the bolt tension and the spacing of bolts. This idealization is illustrated in Figure 21.

(c) Untensioned grouted rock bolts may be idealized as one-dimensional bar elements with material properties similar to those of rock bolts.

(iii) Shotcrete or Concrete Linings - Shotcrete or concrete linings may be idealized as a plane strain structure as shown in Figure 22. Grouting or back packing behind the lining may be modelled in the analysis with materials with different stiffness.

MODIFICATIONS OF THE EXISTING FINITE ELEMENT COMPUTER PROGRAM

It has been indicated that the computer program developed under Contract No. H0210048 is to be modified for the present contract to include the capability for modelling and analyzing structural support schemes used in the construction and design of tunnels, and excavation techniques and construction sequences used in underground construction. Before modifications were made for the present contract, two improvements were incorporated into the program. These were (i) utilization of elasto-plastic stress-strain relationship to compute the axial stress, and (ii) updating the element stiffness at each load increment to improve convergence.

Computation of Axial Stress

Pariseau (1972), in discussing the paper by Chang, Nair and Karwoski (1972), indicates that the equation employed to compute the axial stress for an elasto-plastic analysis is only valid for a rigid, perfectly plastic material. Re-examination of the formulation

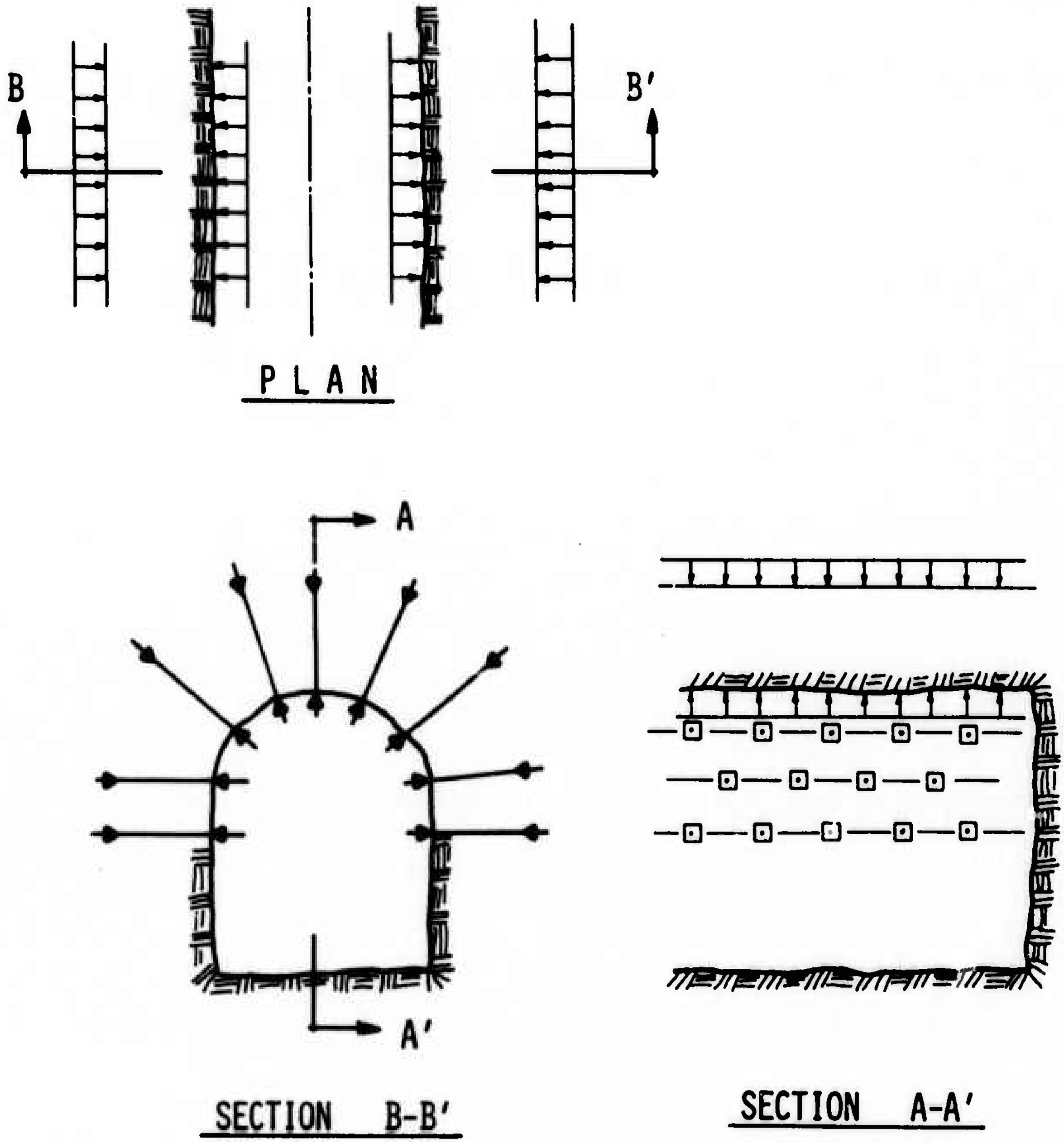


FIGURE 21 - IDEALIZATION OF ROCK BOLTS FOR PLANE ANALYSIS

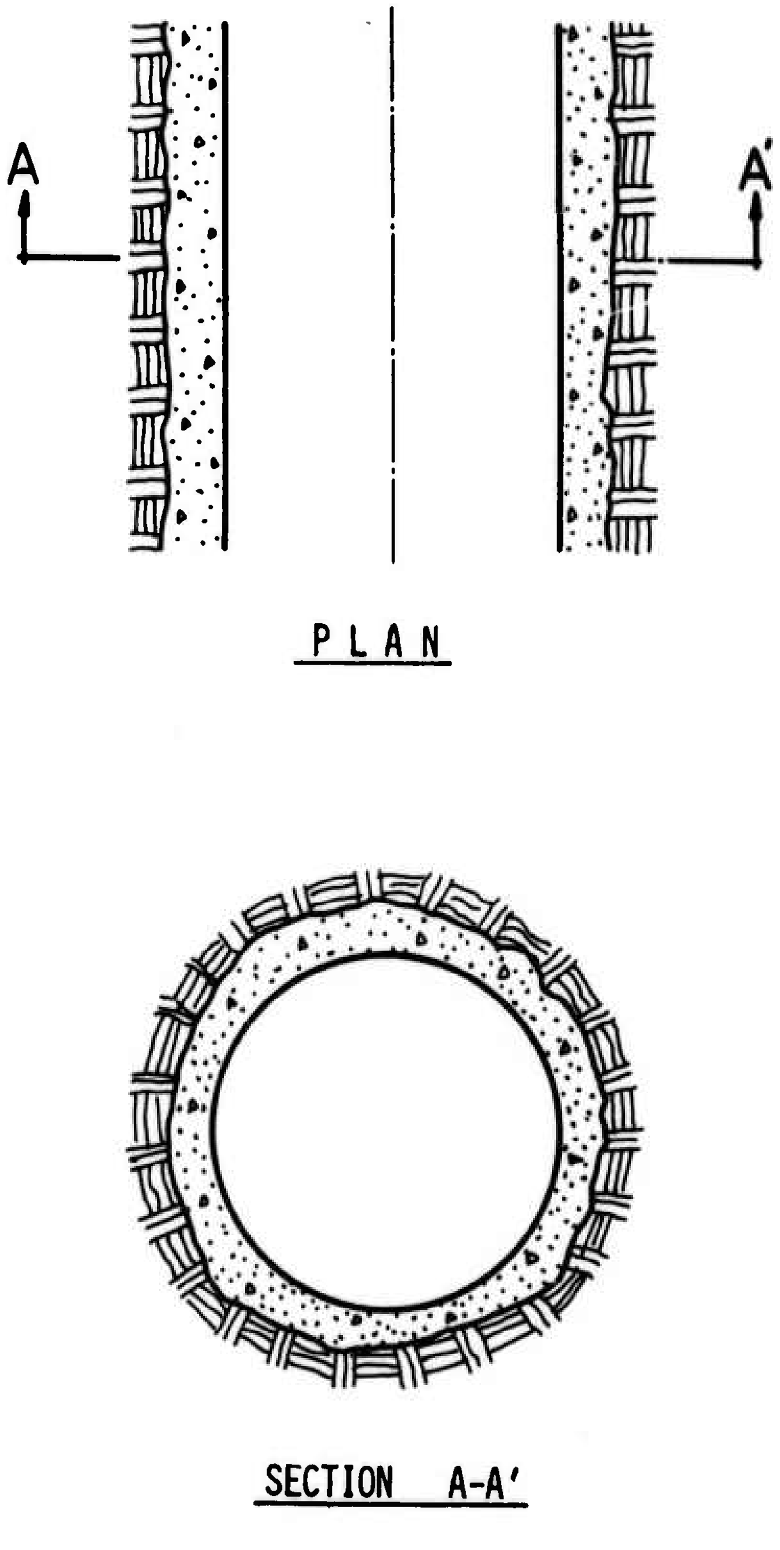


FIGURE 22 - SUPPORT USING CONCRETE OR SHOTCRETE LINING

of the analysis appears to indicate that the results of an elasto-plastic analysis would be correct only if loading is applied in small increments. For each small load increment, each element is checked to determine if the element behaves elastically or plastically. Under the plane strain conditions for which $\epsilon_{zz} = 0$, the increment of the axial stress $\Delta\sigma_{zz}$ is computed depending on the condition whether the element behaves elastically or plastically. When an element goes from an elastic to a plastic state in an increment, an intermediate stress, when yielding commences, is found by interpolation. In this manner, either the material is acting elastically or plastically at any one time, and the axial stress is computed accordingly. Therefore, the axial stress σ_{zz} is correctly calculated at every step of the analysis according to the assumptions made in the analysis. A modification which is described in the following section has been made in the program to compute the axial stress σ_{zz} correctly without any restriction on the magnitude of load increments. The modification is described in the subsequent section.

Incremental Stress-Strain Relations

In the elastic range, the strains are related to the stresses by the generalized Hooke's law under plane strain conditions as

$$\left\{\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \end{bmatrix}\right\} = [D] \left\{\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \end{bmatrix}\right\} \qquad (1)$$

where the strain-stress matrix is

$$[D] = \frac{E}{(1 + \nu)(1 - 2\nu)} \begin{bmatrix} (1 - \nu) & \nu & \nu & 0 \\ \nu & (1 - \nu) & \nu & 0 \\ \nu & \nu & (1 - \nu) & 0 \\ 0 & 0 & 0 & \frac{(1 - 2\nu)}{2} \end{bmatrix} \qquad (2)$$

$\epsilon_z = 0$

and E is the elastic modulus and ν the Poisson's ratio for the linear isotropic elastic material.

In the plastic range, it is assumed that the material behaves perfectly plastically with the yield criteria represented by

$$f = \alpha I_1 + \sqrt{J_2} = k \tag{3}$$

$$\dot{f} = 0 \tag{4}$$

where: α, k = material constants
I_1 = first stress invariant
J_2 = second invariant of stress deviation

The total strain rate ε_{ij} may be expressed by

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{(e)} + \dot{\varepsilon}_{ij}^{(p)} \tag{5}$$

where the elastic strain rate may be computed from the generalized Hooke's law as:

$$\dot{\varepsilon}_{ij}^{(e)} = \frac{1 + \nu}{E} \dot{\sigma}_{ij} - \frac{\nu}{E} \dot{I}_1 \delta_{ij} \tag{6}$$

and the plastic strain rate may be computed from the associated flow rule as:

$$\dot{\varepsilon}_{ij}^{(p)} = \dot{\lambda} \frac{\partial f}{\partial \sigma_{ij}} = \dot{\lambda} \left[\alpha \delta_{ij} + \frac{S_{ij}}{2J_2^{1/2}} \right] \tag{7}$$

where: $\dot{\lambda}$ = a scalar positive function of σ_{ij} and ε_{ij}
δ_{ij} = Kronecker's delta
S_{ij} = stress deviator tensor

Utilizing the preceding relations, Reyes (1966) developed incremental stress-strain relations for an elastic, perfectly plastic material. These relations can be expressed by:

$$\dot{\sigma}_{ij} = 2G\left\{\dot{\varepsilon}_{ij} - \frac{\dot{w}}{kp}\left[h_o\delta_{ij} + \frac{\sigma_{ij}}{2J_2^{1/2}}\right] - \dot{\varepsilon}_{kk}\left[h_2\delta_{ij} + h_1\sigma_{ij}\right]\right\} \tag{8}$$

where: $p = \frac{J_2^{1/2}}{k}\left[1 + 9\alpha^2\frac{K}{G}\right]$

$\dot{w} = \sigma_{ij}\dot{\varepsilon}_{ij}$

for plane strain cases:

$\dot{w} = \sigma_x\dot{\varepsilon}_x + \sigma_y\dot{\varepsilon}_y + \tau_{xy}\dot{\gamma}_{xy}$

K = bulk modulus

G = shear modulus

$$h_o = \frac{3K\alpha}{2G} - \frac{I_1}{6J_2^{1/2}}$$

$$h_1 = h_o \Big/ \left[J_2^{1/2}\left(1 + 9\alpha^2\frac{K}{G}\right)\right]$$

$$h_2 = \frac{2h_o\left[\alpha - \frac{I_1}{6J_2^{1/2}}\right]}{\left(1 + 9\alpha^2\frac{K}{G}\right)} - \frac{3\nu Kk}{EJ_2^{1/2}\left(1 + 9\alpha^2\frac{K}{G}\right)}$$

Equation (8) may be expressed in a matrix form as:

$$\begin{Bmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\sigma}_z \\ \dot{\tau}_{xy} \end{Bmatrix} = [D]_{e.p.} \begin{Bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\varepsilon}_z = 0 \\ \dot{\gamma}_{xy} \end{Bmatrix} \qquad (9)$$

where: $$[D]_{e.p.} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$

$$D_{11} = 2G\left(1 - h_2 - 2h_1\sigma_x - h_3\sigma_x^2\right)$$

$$D_{22} = 2G\left(1 - h_2 - 2h_1\sigma_y - h_3\sigma_y^2\right)$$

$$D_{33} = 0$$

$$D_{44} = 2G\left(\tfrac{1}{2} - h_3\tau_{xy}^2\right)$$

$$D_{12} = D_{21} = -2G\left[h_2 + h_1\left(\sigma_x + \sigma_y\right) + h_3\sigma_x\sigma_y\right]$$

$$D_{13} = D_{31} = -2G\left[h_2 + h_1\left(\sigma_x + \sigma_z\right) + h_3\sigma_x\sigma_z\right]$$

$$D_{14} = D_{41} = -2G\left[h_1\tau_{xy} + h_3\sigma_x\tau_{xy}\right]$$

$$D_{23} = D_{32} = -2G\left[h_2 + h_1\left(\sigma_y + \sigma_z\right) + h_3\sigma_y\sigma_z\right]$$

$$D_{24} = D_{42} = -2G\left[h_1\tau_{xy} + h_3\sigma_y\tau_{xy}\right]$$

$$D_{34} = D_{43} = -2G\left[h_1\tau_{xy} + h_3\sigma_z\tau_{xy}\right]$$

and $$h_3 = \frac{1}{2J_2\left(1 + 9\alpha^2\frac{K}{G}\right)}$$

The strain-stress relation described by equations (1) or (9) has been used in the program to compute the axial stress and to form the stiffness depending whether the element is in the elastic or plastic range.

Updating the Element Stiffnesses

It has been experienced that in an elasto-plastic analyses, if a constant initial stiffness is used in the initial stress approach, the computing time is greatly reduced for each iteration. However, it has been shown that in this case, the solution convergence is very slow. The program has been modified to improve the rate of the solution convergence. An additional option has been added to the program so that the stiffness of the system may be updated at each new increment of load. It has been found that by doing this, generally 2 to 4 iterations are sufficient at each load increment to ensure that the equilibrium conditions are satisfied.

The technique for performing nonlinear analysis with the above modification is illustrated in Figure 23. This technique may be summarized as follows:

(1) For each increment of load, an initial elastic stiffness is used and the elastic solution is obtained. Using the elastic stiffness for each increment of load may ensure that a correct solution is obtained if the structure is unloaded.

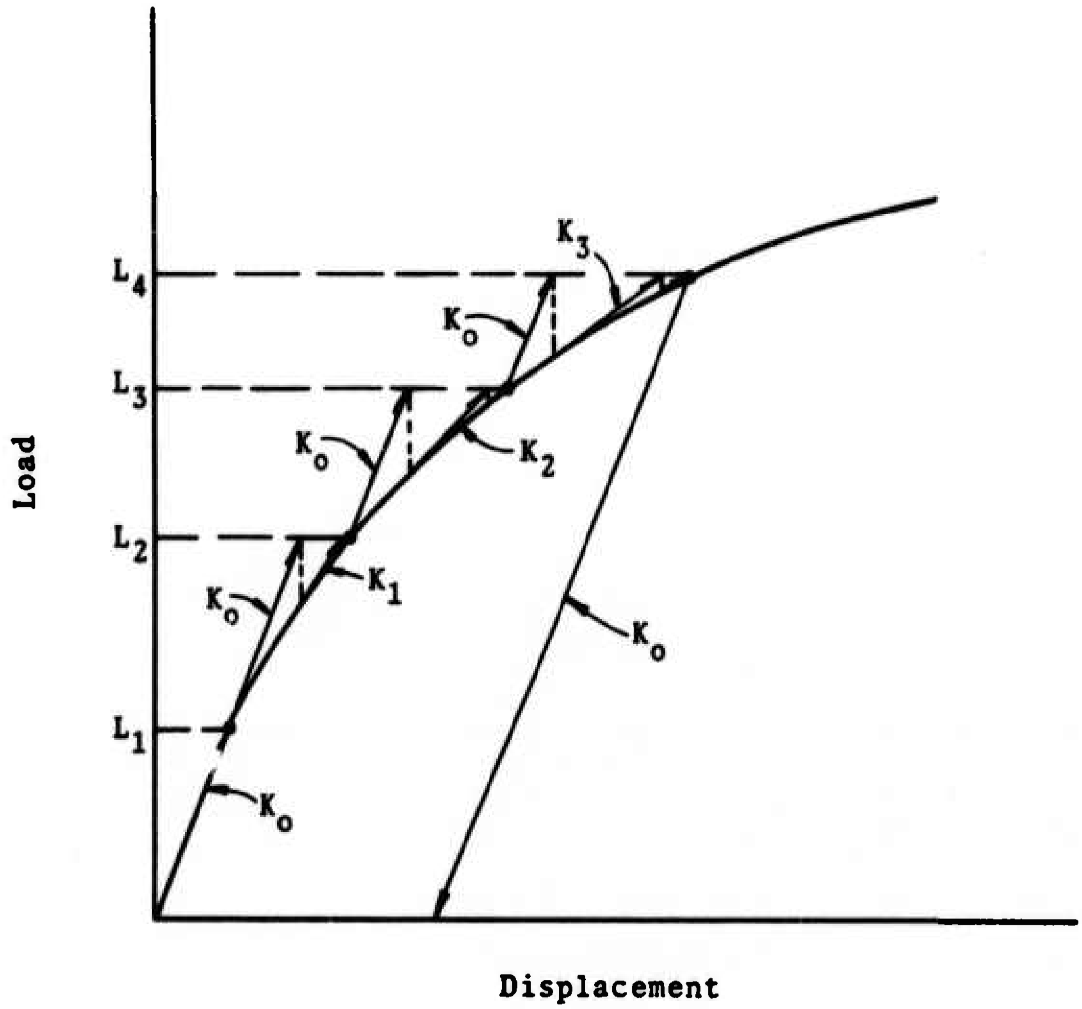


FIGURE 23 - MODIFIED TECHNIQUE FOR NONLINEAR ANALYSIS

(2) If the elastic solution indicates that some elements are in yield, excess stresses are computed and redistributed by iterative processes. During the redistribution of excess stresses, a new stiffness updated after the first iteration is used in subsequent iterations.

(3) Step (2) is repeated until the equilibrium conditions are satisfied.

(4) Repeat Steps (1) to (3) for all increments of load.

A flow diagram illustrating the algorithm for the modified elasto-plastic analysis is shown in Figure 24. It may be noted that the incremental stress-strain relations described in the previous section have been incorporated in the formulation of the method of analysis.

<u>Modifications of the Computer Program for Modelling Excavation Techniques, Structural Support Schemes and Construction Sequences</u>
The general concepts in developing computational models for simulating excavation techniques, structural support schemes, and excavation and construction sequences have been described in the previous sections. The detailed procedure can be summarized as follows:

(1) For illustrative purposes, an underground opening is shown in Figure 25 to be excavated in several stages. The first step in the analysis is to assign values of initial stresses, σ_i, to each element. The initial state of stress may be estimated or determined in the field.

(2) Read in data describing the current stage of construction, i.e., elements to be excavated, elements situated within disturbed zones, nodal forces for simulating rock bolts installation, and/or elements as a structural support or lining, if any;

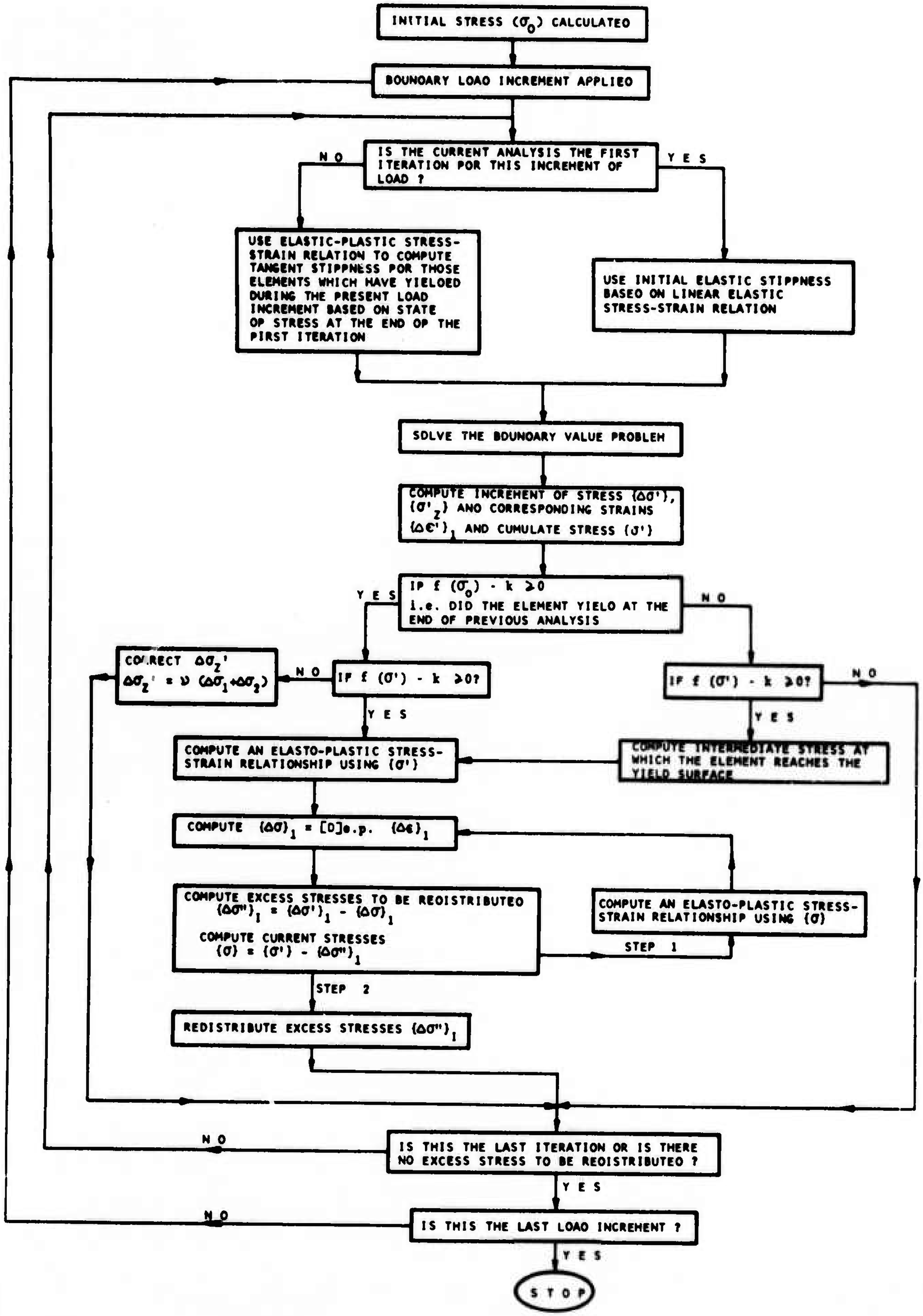


FIGURE 24 - MODIFIED STRESS TRANSFER TECHNIQUE FOR ELASTO-PLASTIC ANALYSIS

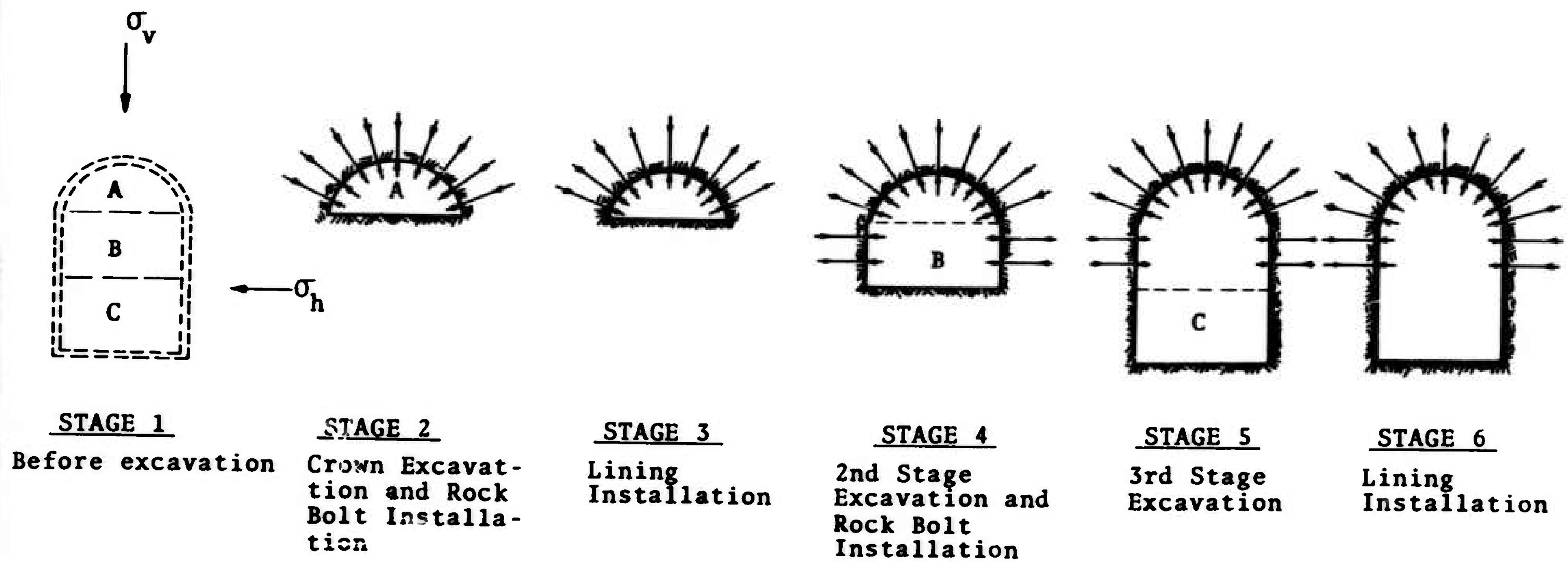


FIGURE 25 - A SCHEMATIC DIAGRAM SHOWING EXCAVATION, CONSTRUCTION SEQUENCES AND GROUND SUPPORT INSTALLATIONS DURING CONSTRUCTION OF AN UNDERGROUND OPENING

nodal points along the excavated face which describe the geometry of the current stage of excavation. The elements to be excavated are assigned a small modulus to simulate the existence of a cavity. The elements in the disturbed zones are assigned with a lower modulus or no tensile strength. The elements for structural supports are assigned with appropriate properties corresponding to the material used for the structural supports.

(3) Initial stresses, σ_i, on the boundary exposed by excavation are computed from stresses in the surrounding elements using a technique similar to that used by Clough and Duncan (1969). The detailed procedure has been described in the final report under Contract No. HO210046. To simulate excavation, changes in stress, $\Delta\sigma$, which are equal in magnitude and opposite in sign to the initial stresses, σ_i, are applied to the boundary exposed by excavation.

(4) An initial elastic analysis is conducted. Increments of elastic stresses and strains are computed.

(5) Unbalanced excess stresses are determined and redistributed by iterative processes until the equilibrium conditions are satisfied.

(6) Repeat Steps (2) to (5) for all construction stages.

A flow diagram showing the proposed procedure to simulate excavation techniques, structural support schemes, and excavation and construction sequences is illustrated in Figure 26. The proposed procedure has been used to modify the existing computer program. To verify and illustrate the use of the modified computer program, several example problems were analyzed. The results of the analyses are described in the following section.

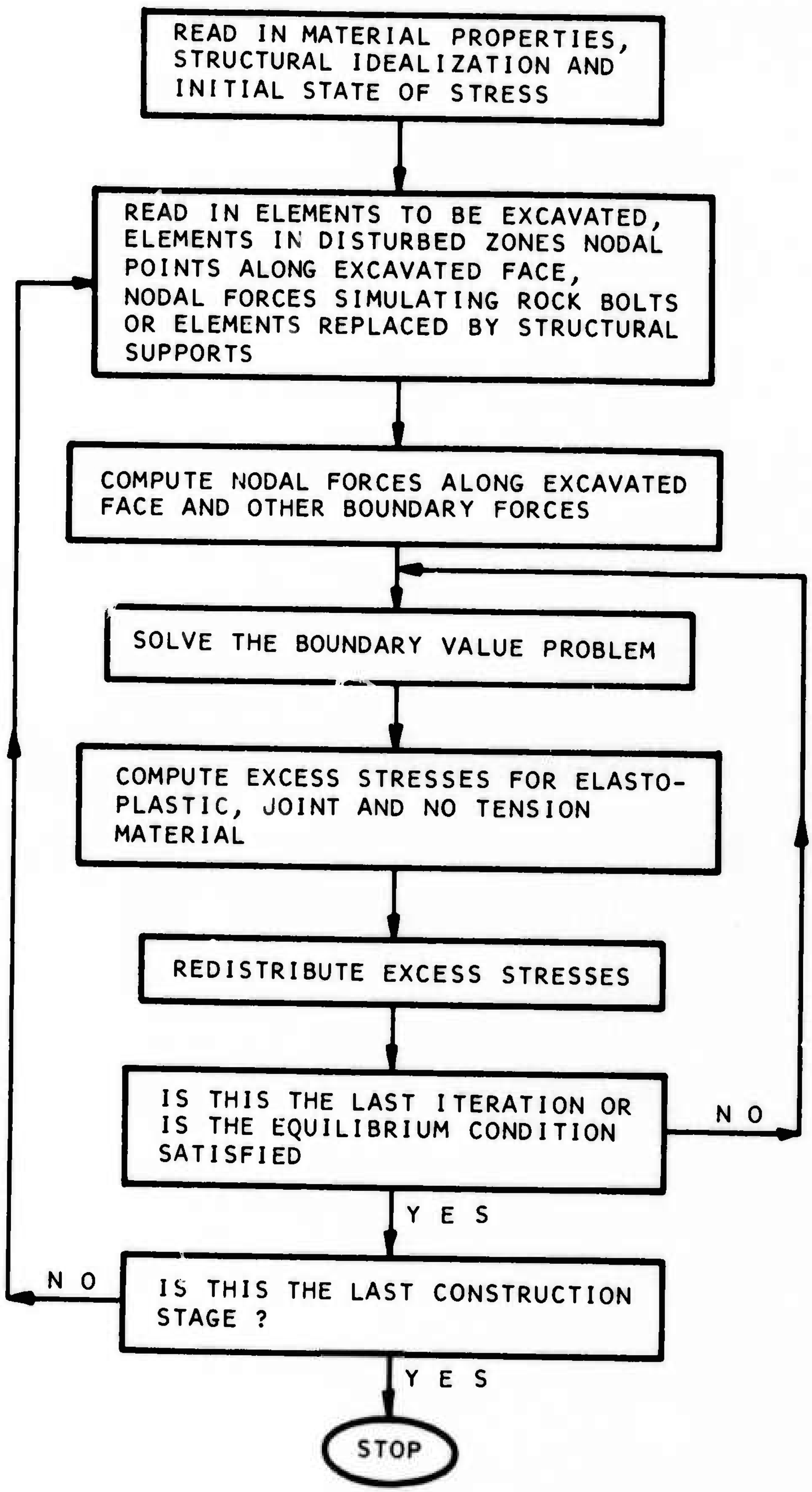


FIGURE 26 - A PROCEDURE FOR SIMULATING EXCAVATION AND CONSTRUCTION SEQUENCES

ILLUSTRATIVE PROBLEMS

Definition of Problems

The following four example problems were analyzed using the modified computer program:

I. Elasto-plastic analysis of a thick-walled circular tube with the Von Mises yield criterion. A closed form solution is available for this case for verification.

II. Elasto-plastic analysis of a circular opening with the generalized Mohr-Coulomb yield criterion. The results are compared with those obtained by Reyes (1966).

III. Elastic analysis of a circular opening by two stages of excavation and gravity turn-on procedures.

IV. Analysis of a circular opening reinforced by rock bolts and a concrete lining.

Problems (I) and (II) were analyzed to show the improved accuracy and rate of the solution convergence as compared with those reported in the final report under Contract No. H0210046. Problem (III) was analyzed to indicate the ability of the computational technique used to simulate excavation sequences. Problem (IV) was analyzed to illustrate capabilities of the modified computer program for simulation of excavation techniques, support installation and construction sequences.

Results

I. Elasto-plastic Analysis of a Thick-walled Circular Tube Subject to Internal Pressure

The dimensions of the rube, the material properties and the finite element idealization of the problem are shown in Figure 27. The results of the analysis, together with the closed form solution

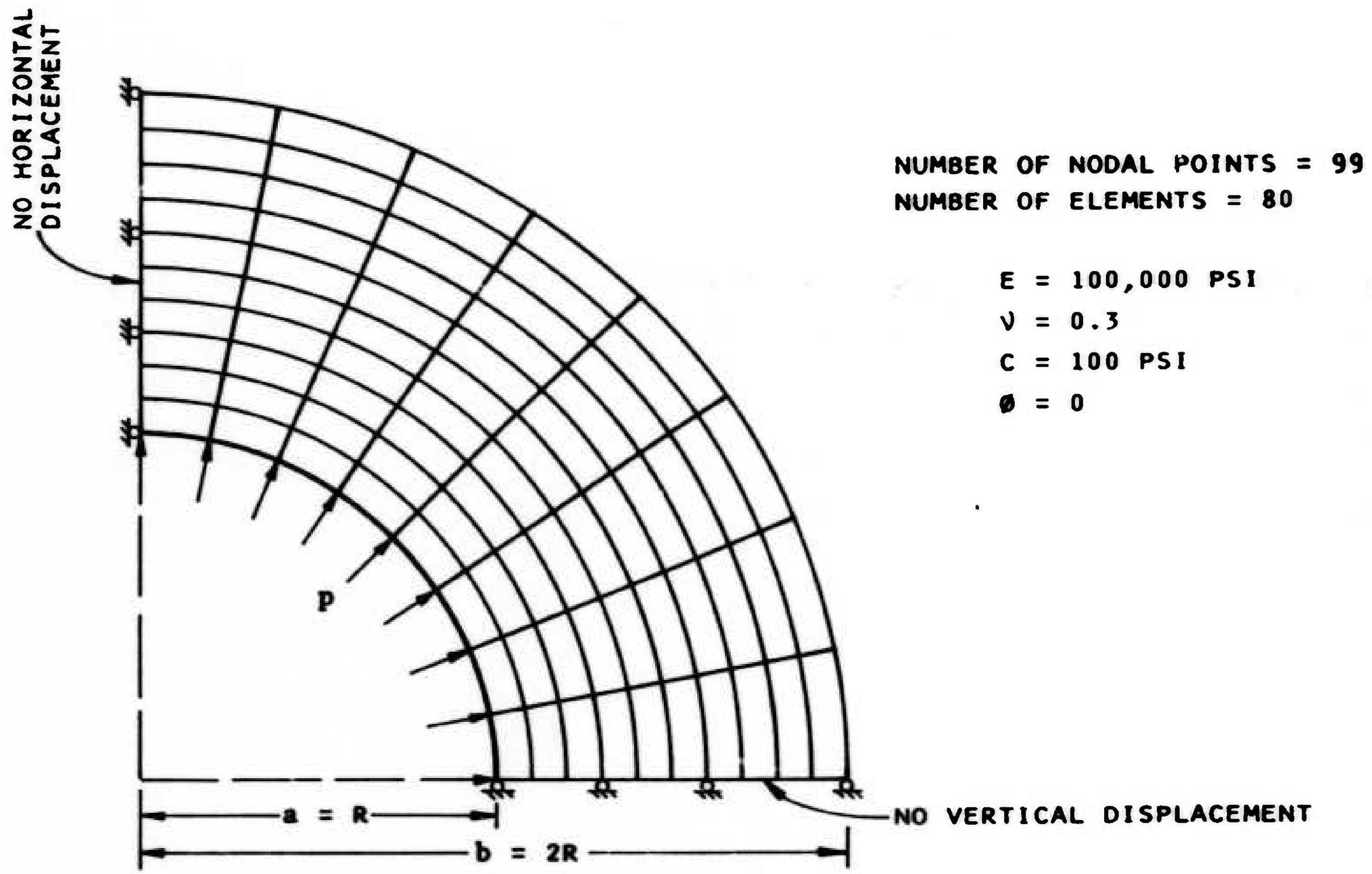


FIGURE 27 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A THICK-WALLED CIRCULAR TUBE (b = 2a)

obtained by Prager and Hodge (1951) are shown in Figures 28 through 31. Comparison between the results obtained from the finite element analysis and those from the closed form solution indicates good agreement.

II. Elasto-plastic Analysis of a Circular Opening with the Generlized Mohr-Coulomb Yield Criterion

The finite element idealization together with the definitions of the problem is shown on Figure 32. The analysis was conducted by applying pressures on the cavity face. The boundary pressures were applied in five increments. It should be noted that only two to four iterations were required for each increment of load for solution convergence, indicating improved rates of solution convergence obtained by updating the stiffness, the additional option added to the existing program. The results of the analysis together with those obtained by Reyes (1966) are shown in Figures 33 and 34.

III. Elastic Analysis of a Circular Opening by Two-Stage Excavation and Gravity Turn-on Procedures

The finite element idealization of a circular opening 20 feet in diameter is shown on Figure 35. The elastic stress distribution was obtained by a two-stage excavation procedure. The initial state of stress was obtained by the gravity turn-on procedure. The first-stage excavation of a 10-foot-diameter cavity was conducted using the proposed technique for simulating excavation. After the elastic stress distribution for the first-stage excavation was obtained, the remaining rock was removed by the second-stage excavation. The elastic stress distribution obtained by the two-stage excavation procedure is compared with the one-step gravity turn-on procedure as shown in Figure 36. It may be noted that the virtually identical stress distribution was obtained by both procedures indicating that the proposed technique for simulating excavation is accurate enough for engineering purposes.

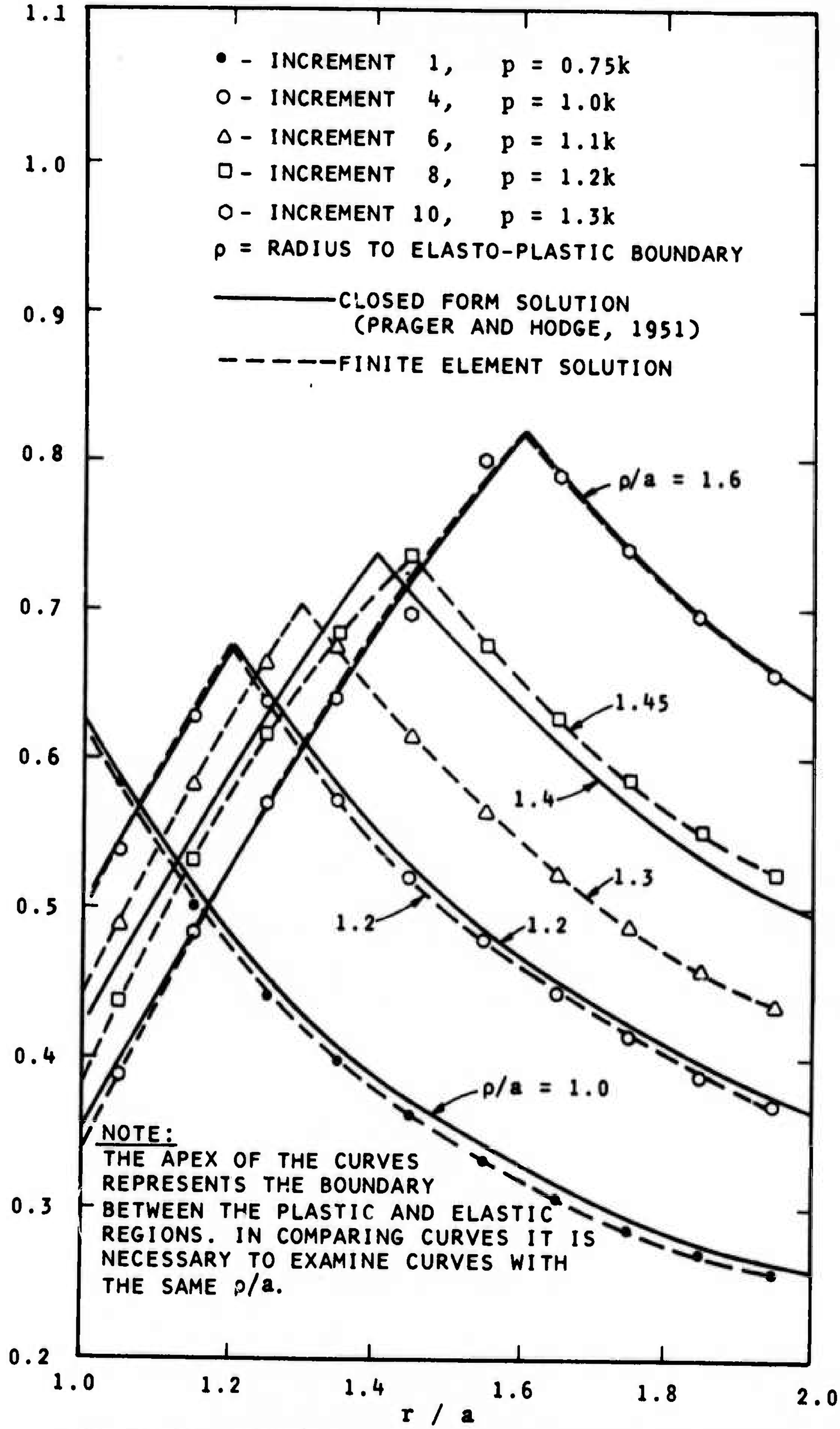


FIGURE 28 - DISTRIBUTION OF CIRCUMFERENTIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

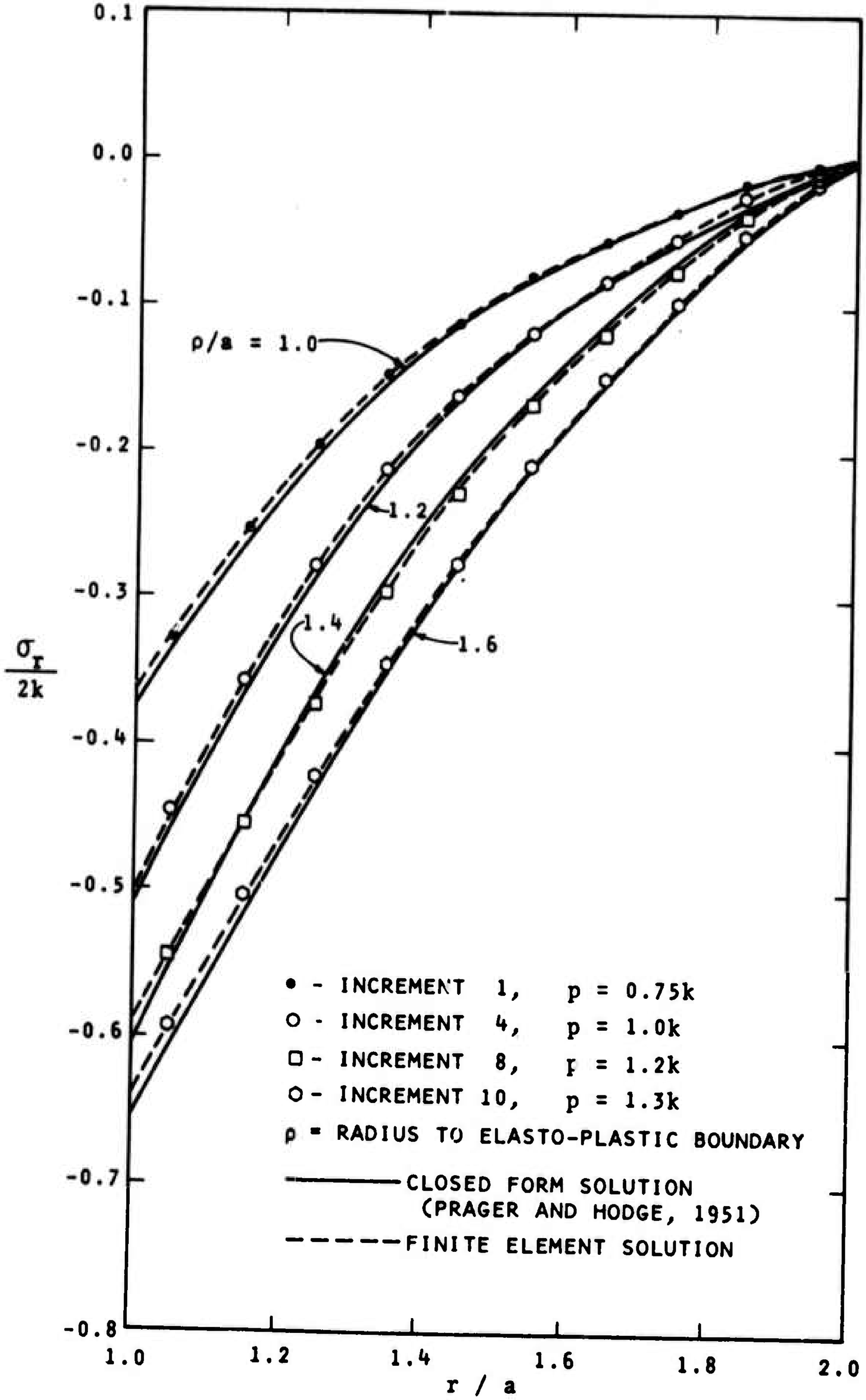


FIGURE 29 - DISTRIBUTION OF RADIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

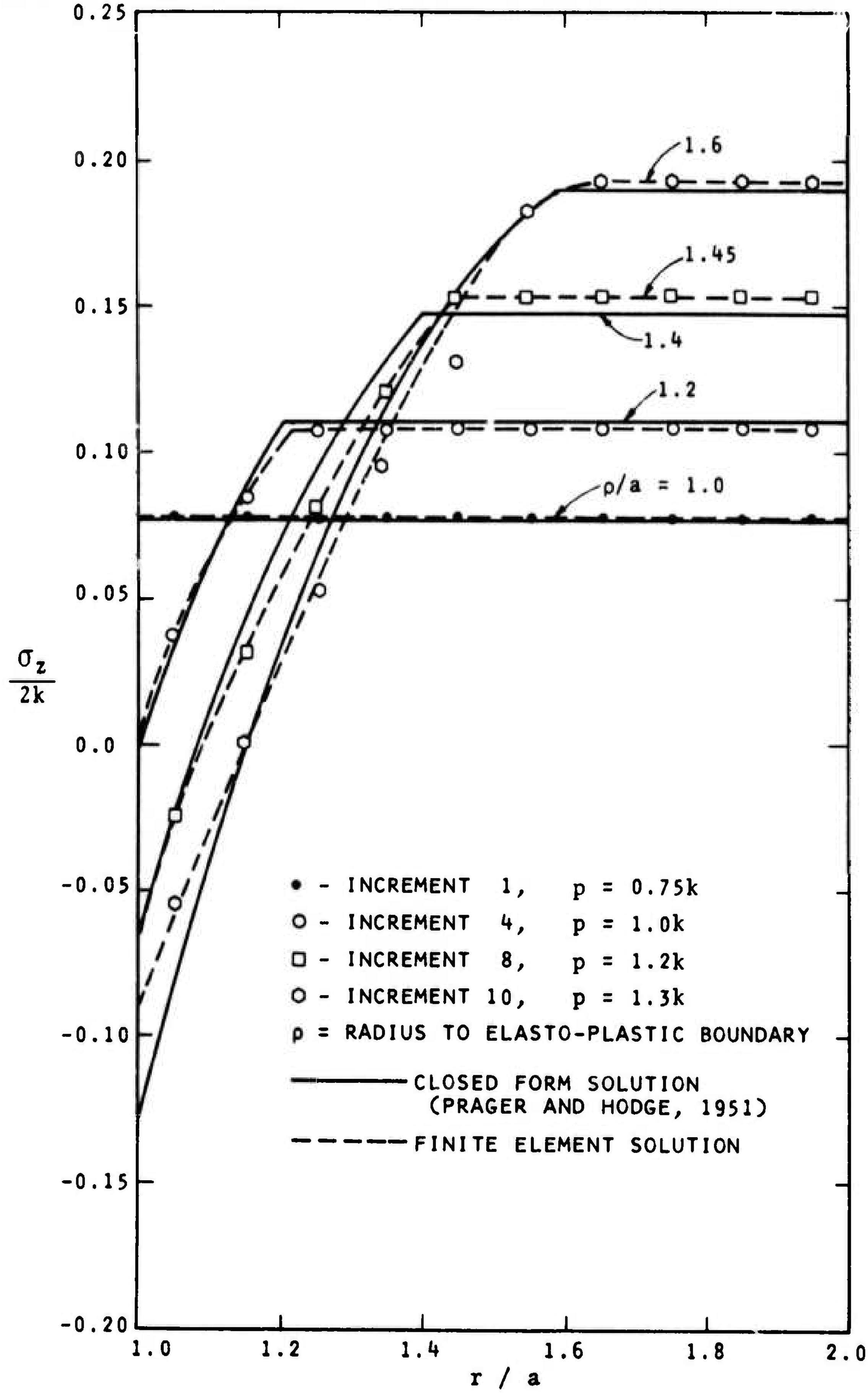


FIGURE 30 - DISTRIBUTION OF AXIAL STRESS FOR A THICK-WALLED CIRCULAR TUBE

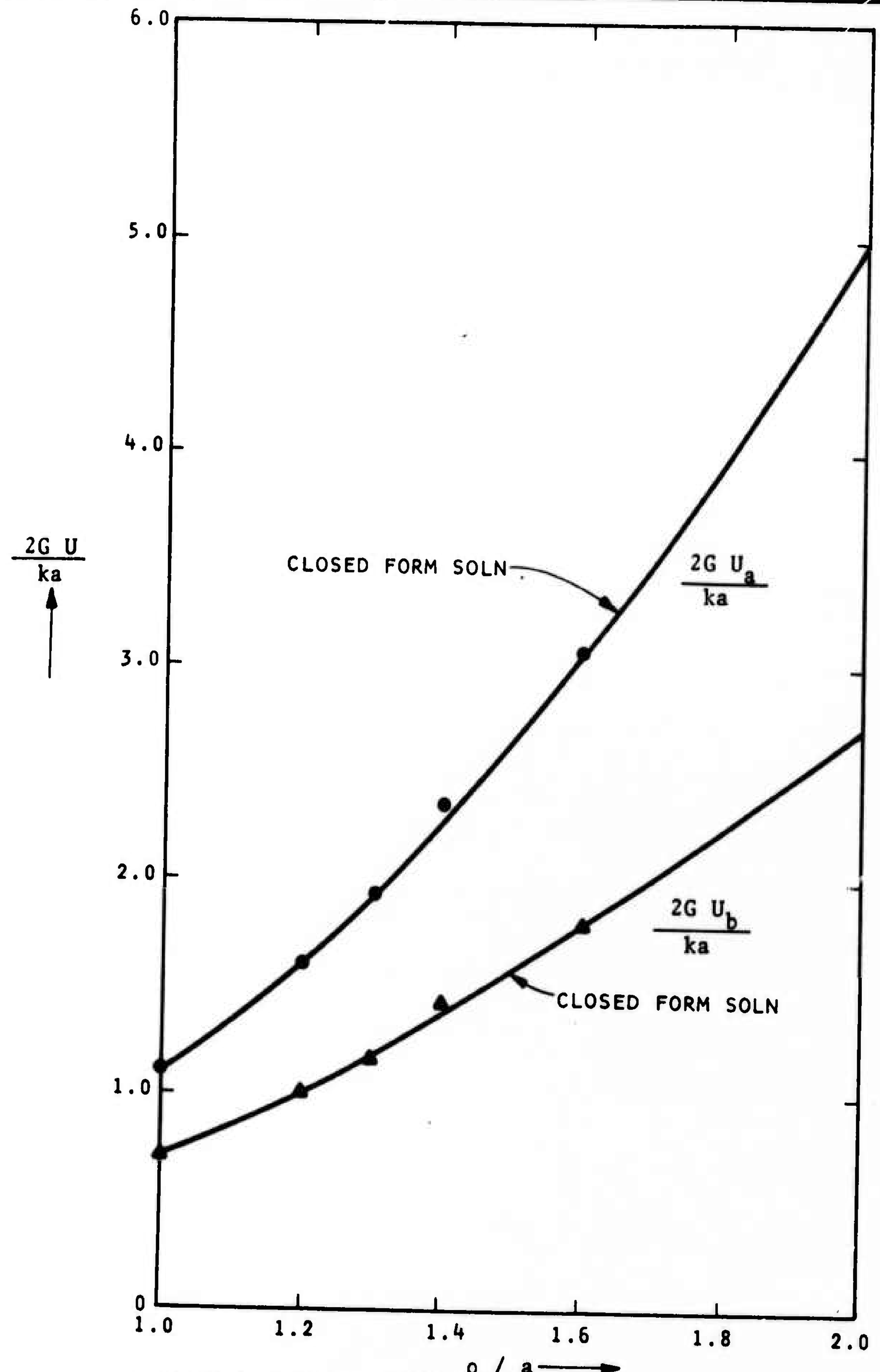
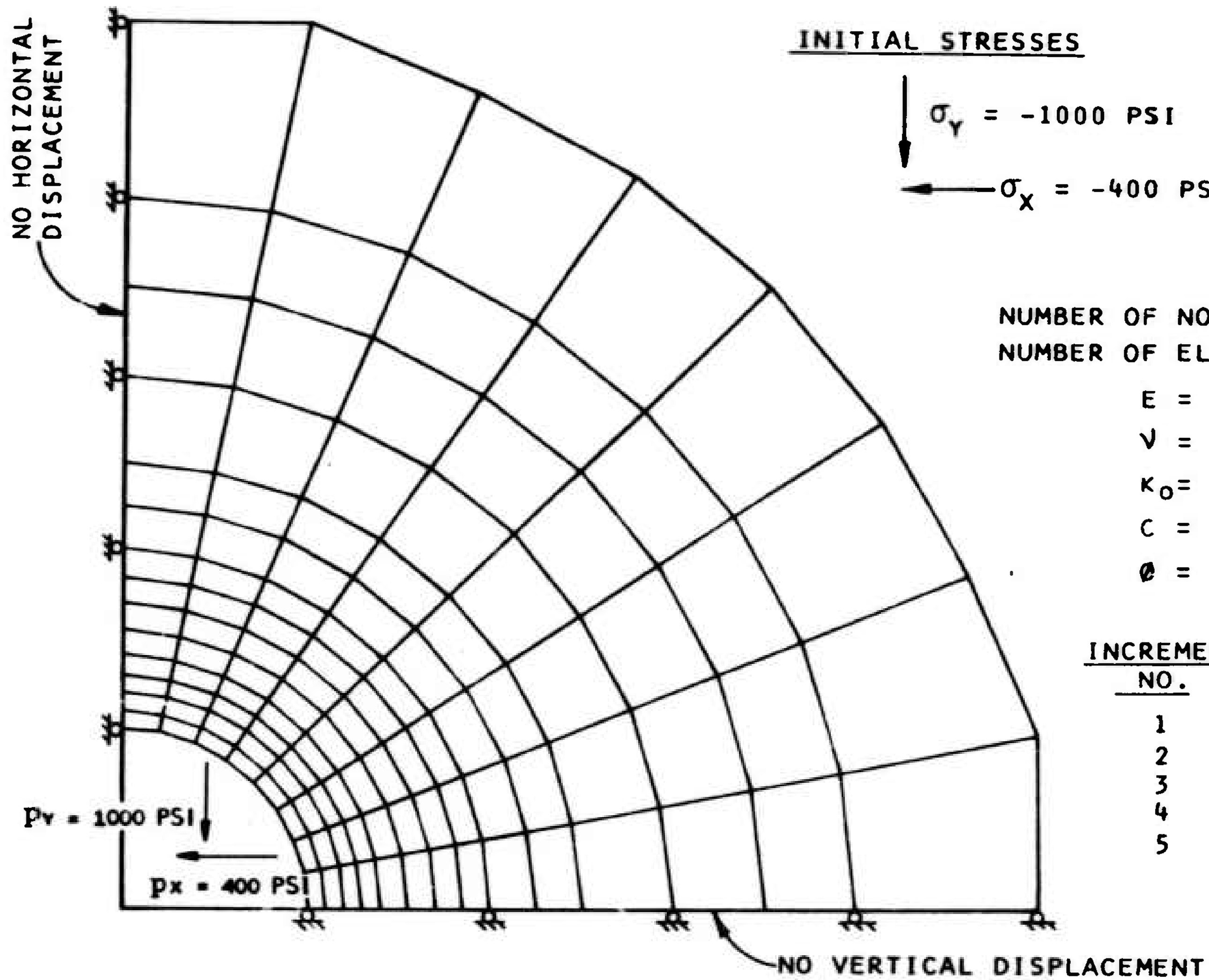


FIGURE 31 - RADIAL DISPLACEMENT U_a, U_b VS. RADIUS ρ OF ELASTIC-PLASTIC BOUNDARY FOR A THICK-WALLED CIRCULAR TUBE



INITIAL STRESSES

σ_Y = -1000 PSI

σ_X = -400 PSI

0 — 10 FT.
SCALE

NUMBER OF NODAL POINTS = 135
NUMBER OF ELEMENTS = 112

E = 72,000 KSF

ν = 0.2

K_o = 0.4

C = 40.32 KSF

ϕ = 30°

INCREMENT NO.	PERCENTAGE OF TOTAL BOUNDARY PRESSURE
1	0.370
2	0.256
3	0.194
4	0.100
5	0.08

FIGURE 32 - FINITE ELEMENT MESH FOR AN ELASTO-PLASTIC ANALYSIS OF A CIRCULAR OPENING

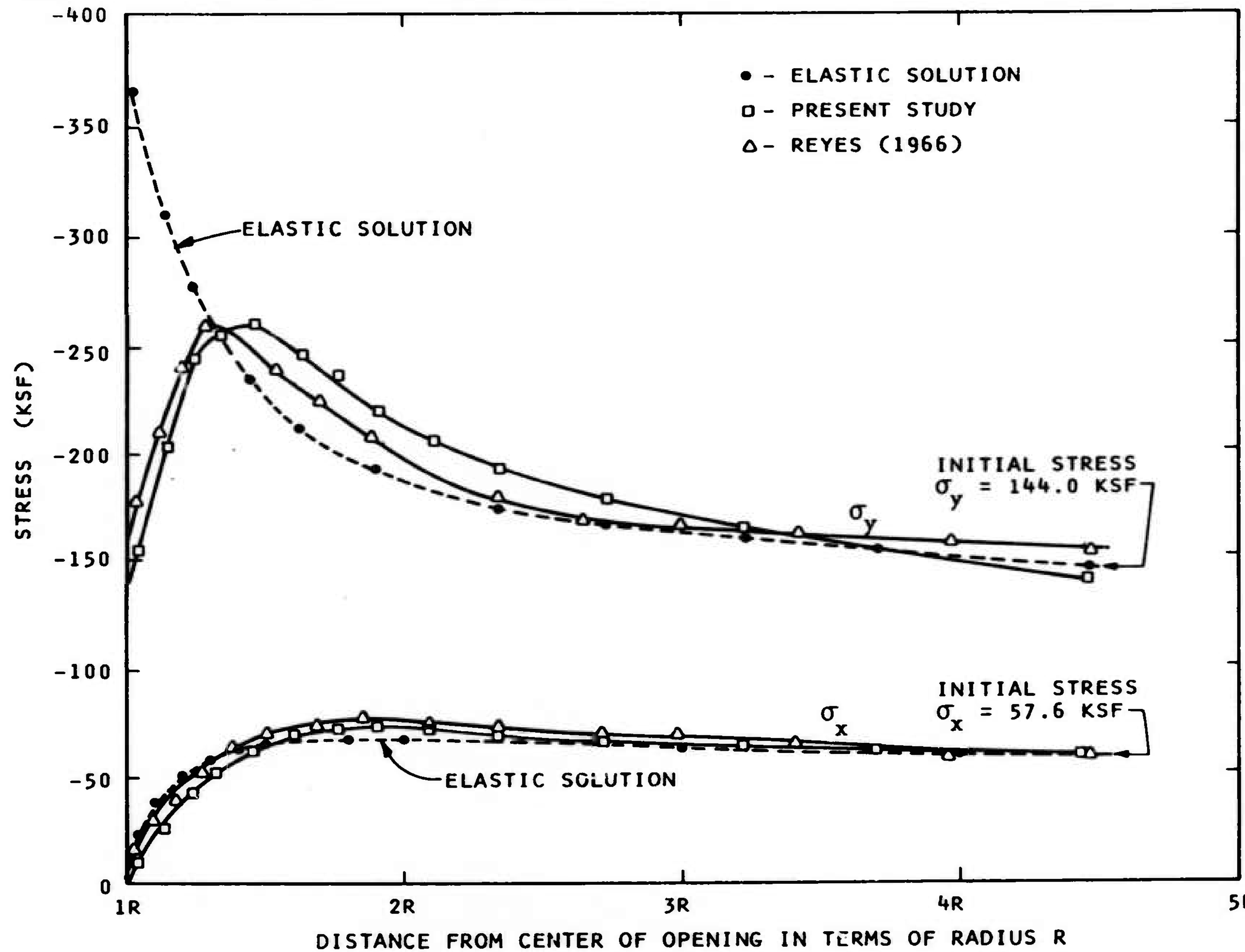


FIGURE 33 - VERTICAL AND HORIZONTAL STRESSES ALONG HORIZONTAL SECTION OF A CIRCULAR OPENING

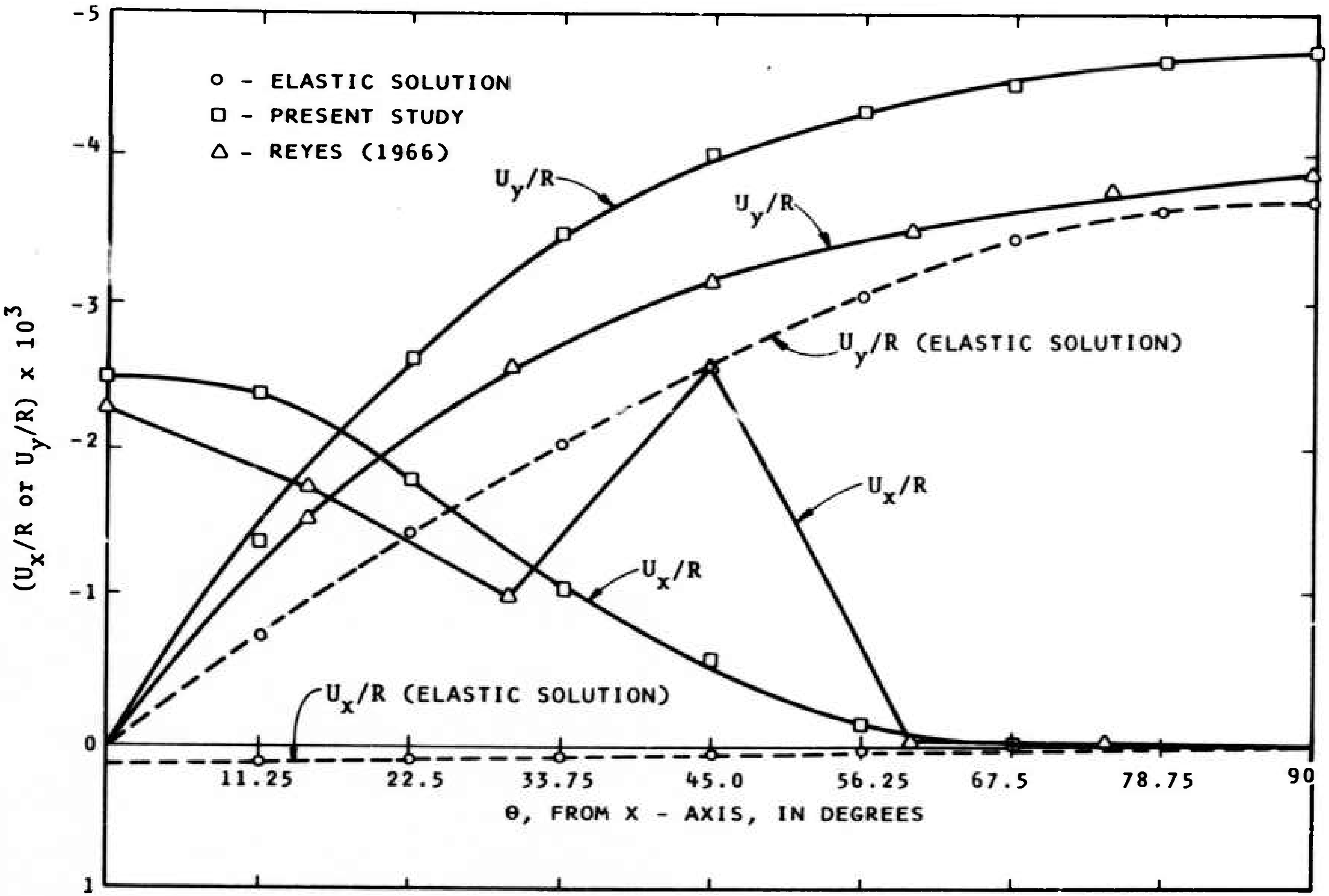


FIGURE 34 - DEFORMATION ALONG CAVITY FACE OF A CIRCULAR OPENING AS COMPUTED BY ELASTIC AND ELASTIC-PLASTIC ANALYSIS

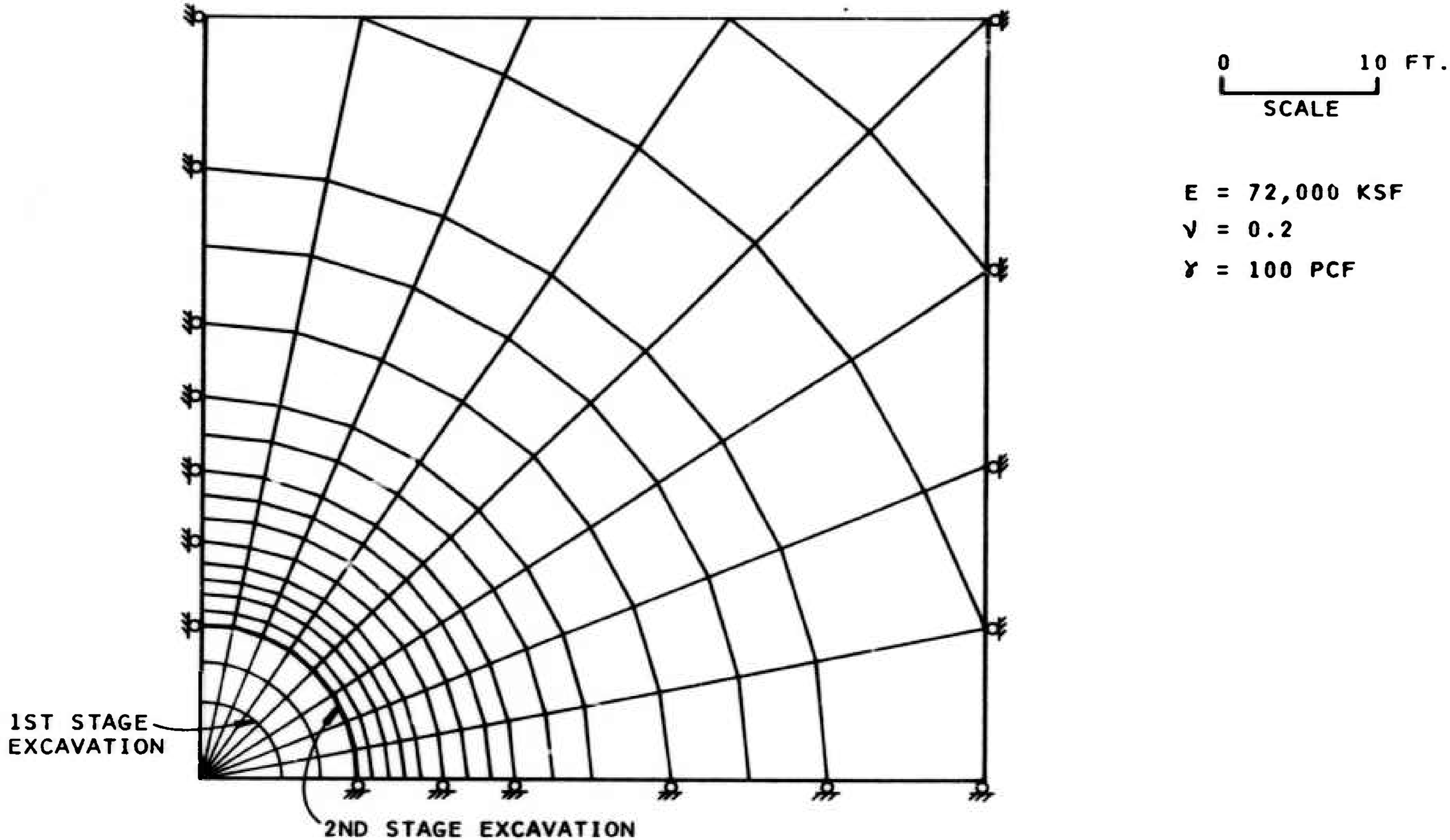


FIGURE 35 - FINITE ELEMENT MESH FOR LINEAR ELASTIC ANALYSIS OF A CIRCULAR OPENING BY TWO-STAGE EXCAVATION

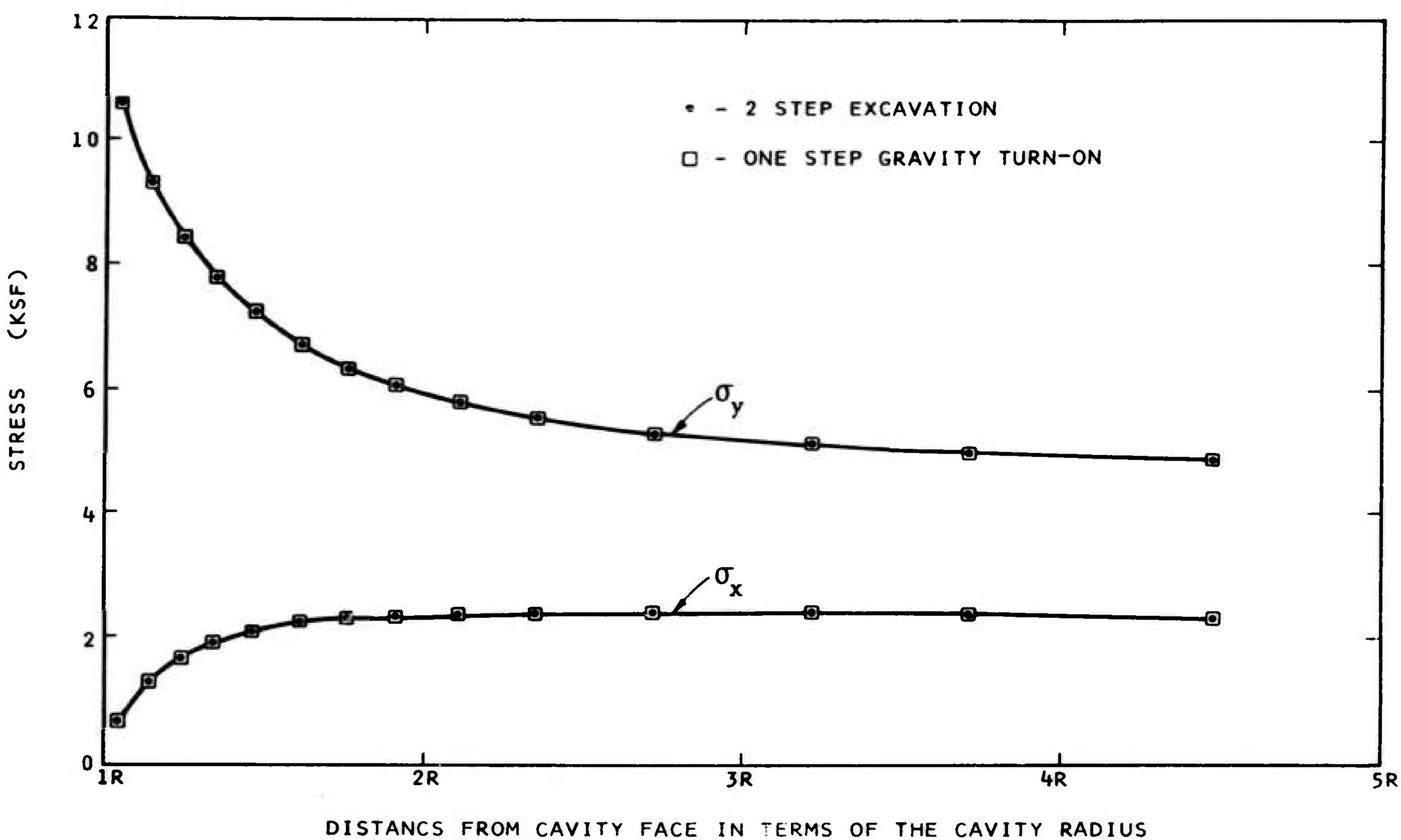


FIGURE 36 - COMPARISON OF VERTICAL AND HORIZONTAL STRESSES ALONG HORIZONTAL SECTION OF A CIRCULAR OPENING OBTAINED BY TWO-STAGE EXCAVATION AND BY GRAVITY TURN-ON ANALYSIS (ELASTIC)

IV. Analysis of a Circular Opening Reinforced by Rock Bolts and a Concrete Lining

The assumed construction sequences of the opening analyzed is shown in Figure 37. It is assumed that the opening is to be excavated full face, followed by installation of a set of rock bolts. At the final stage of construction, a concrete liner, 1 foot thick, is installed. The opening, 10 feet in diameter, is assumed to be situated in a rock mass under an initial stress field which consists of a vertical stress of 1000 psi and a horizontal stress of 400 psi. The finite element idealization of the problem is shown in Figure 38. Sixteen sets of rock bolts are installed. Each set is tensioned to 28,300 pounds per linear foot along the tunnel axis equivalent to 100 psi compressive stress applied to the cavity face. The analysis was conducted in accordance with the following procedure: (1) an elastic analysis of the structure subjected to boundary pressures simulating excavation and installation of rock bolts was first performed; (2) a concrete liner was then "installed." The results of analysis obtained from Step (1) was considered as an initial condition in the subsequent analyses in which excess stresses were redistributed. The results of the analysis, as shown in terms of the stress distribution in both the concrete liner and the rock surrounding the opening, is illustrated in Figure 39. For the purpose of comparison, the results of the analysis on the unsupported opening obtained in Problem (II) are also shown in Figure 39. The difference in the stress distribution obtained in Problems (II) and (IV) is due to the installation of the rock bolts and the concrete liner.

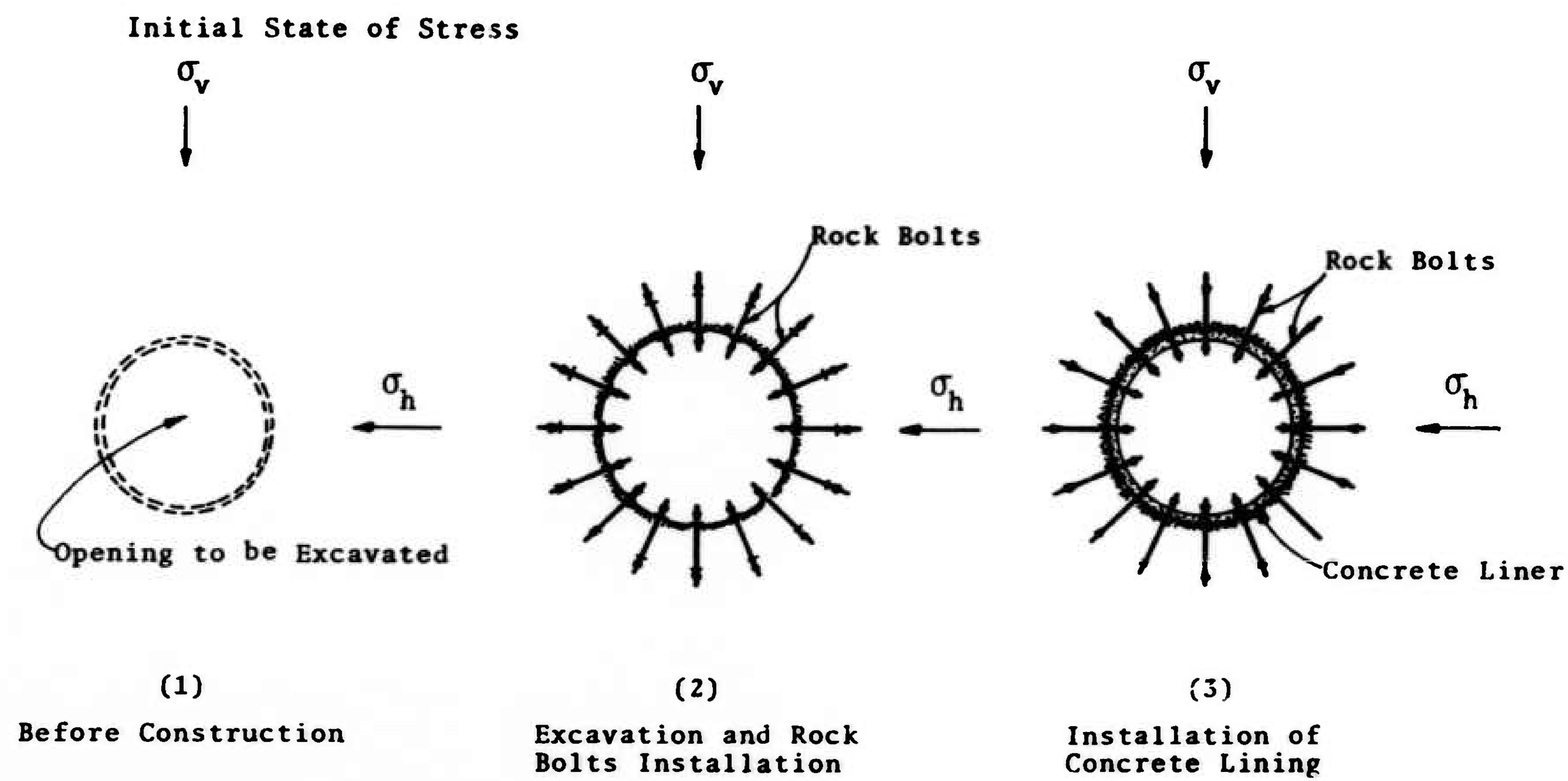


FIGURE 37 - ASSUMED CONSTRUCTION SEQUENCES OF A CIRCULAR OPENING REINFORCED BY ROCK BOLTS AND A CONCRETE LINING

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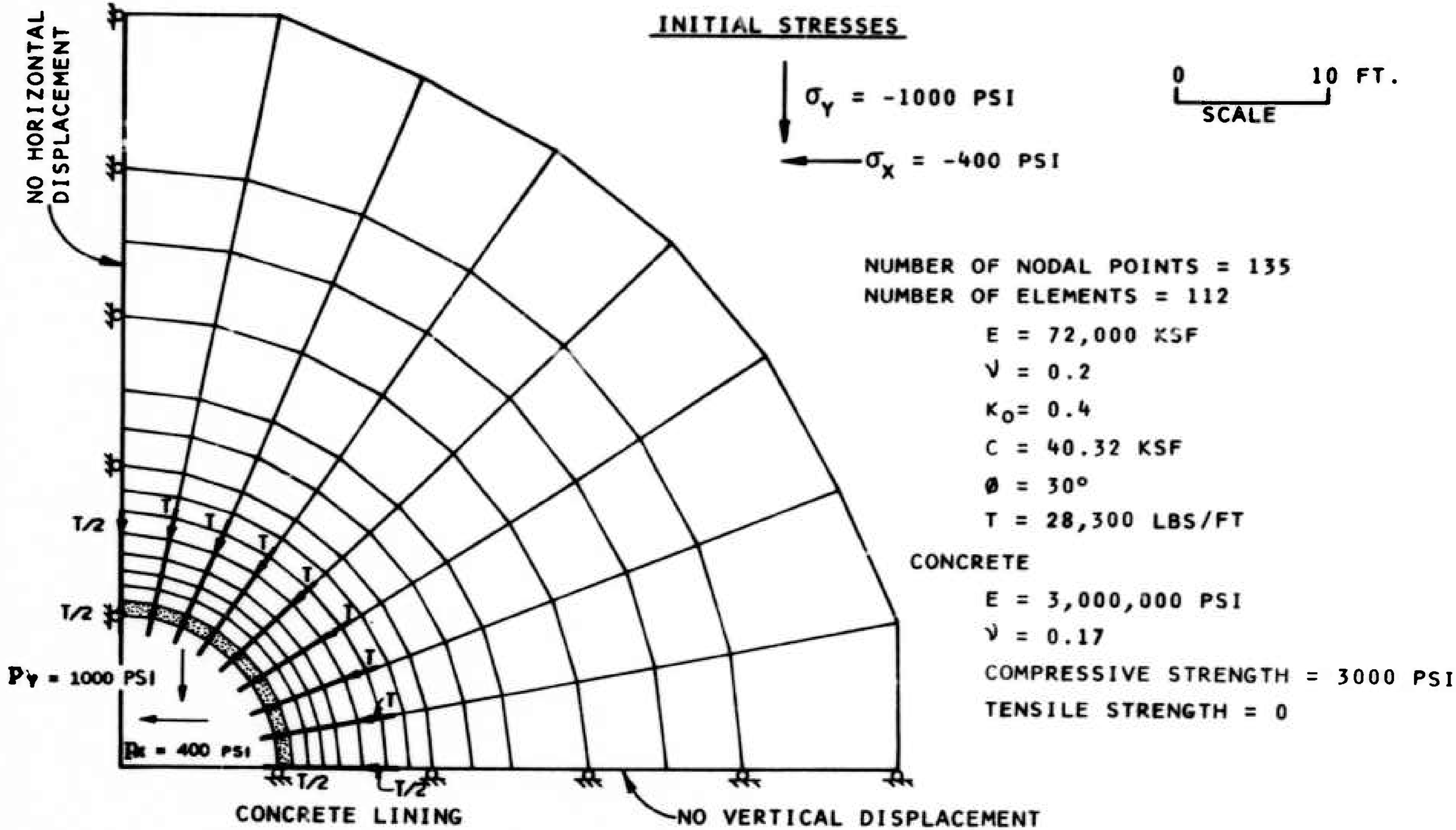


FIGURE 38 - FINITE ELEMENT MESH FOR ANALYSIS OF A CIRCULAR OPENING REINFORCED BY ROCK BOLTS AND CONCRETE LINING

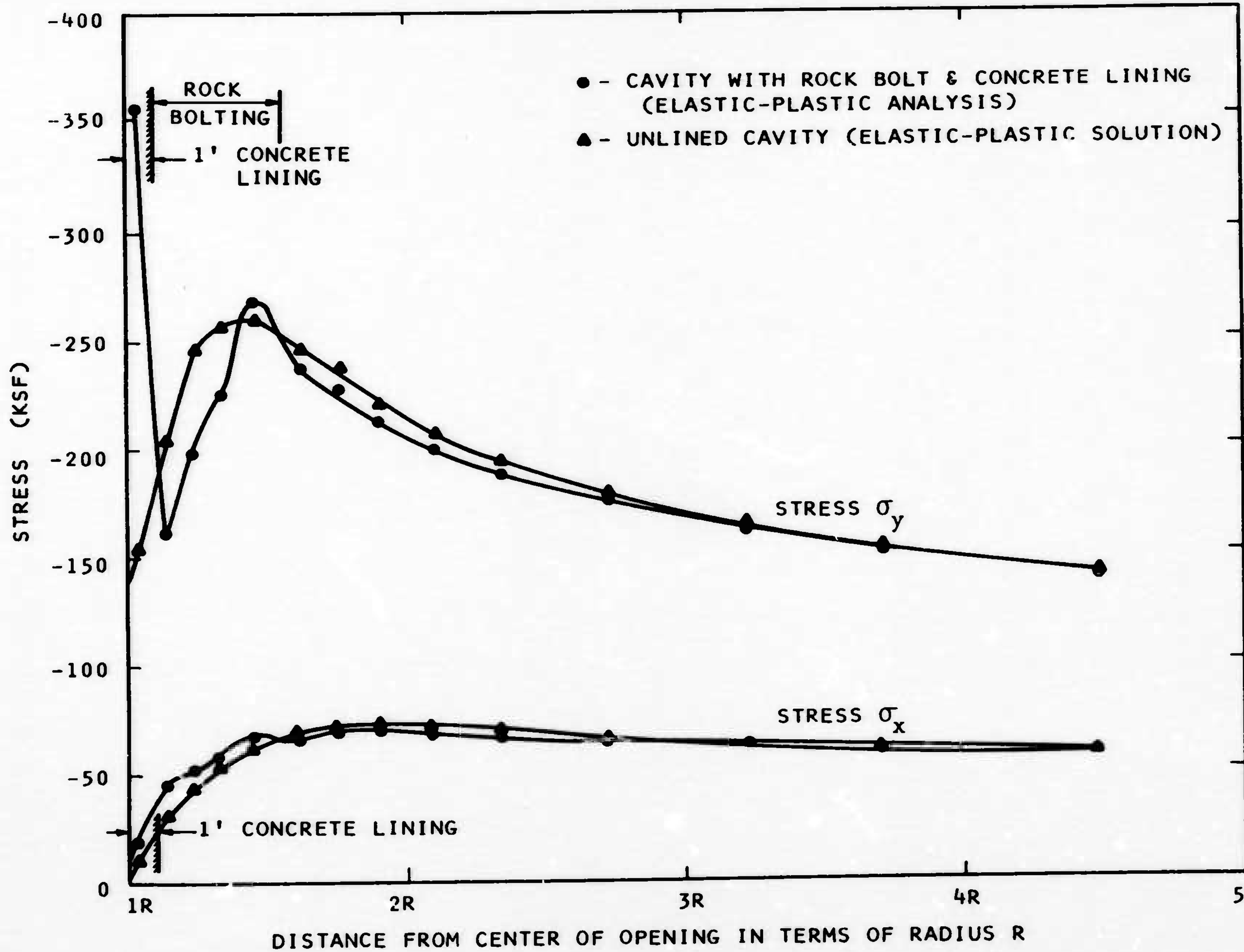


FIGURE 39 - VERTICAL AND HORIZONTAL STRESSES ALONG HORIZONTAL SECTION OF A CIRCULAR OPENING REINFORCED BY ROCK BOLTS AND CONCRETE LINING

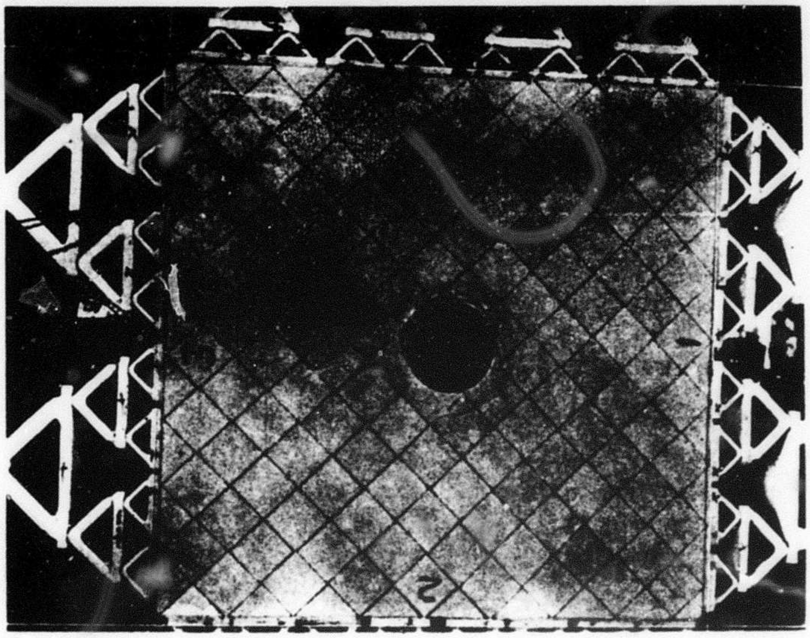
EVALUATION OF ANALYTICAL METHOD (COMPUTER PROGRAM) - CASE HISTORY STUDIES

To evaluate the capabilities and reliability and illustrate the use of the computer code developed, studies were made on a number of well-documented model tests and case histories of underground openings. These include the analysis of model tests on lined and unlined openings in jointed rock, the analysis of Tumut I Underground Power Station, and a rock tunnel of Washington D.C. Metro. Analyses of these case history studies are described in the following sections.

ANALYSIS OF LABORATORY MODEL OF LINED AND UNLINED OPENINGS IN JOINTED ROCK

Description of Model Study

Hendron, et al. (1972) conducted a series of model tests on lined openings in jointed rock fabricated by a rock-like material. One of these models analyzed in this study is shown in Fig. 40. The joint blocks used to construct the model were made by sawing them out of larger compacted blocks. The material used was a water/plastic/sand mixture and was the same as that used by Heuer and Hendron (1969) in model tests of unlined openings in solid blocks. The model analyzed in this study was constructed with a 2-inch joint spacing in two mutually perpendicular directions at 45° to the principal loading directions. A 4-inch-diameter opening was cored after the model was constructed in the testing machine and a seating load of about 25 psi was applied in both the vertical and horizontal directions. The 0.035-inch-thick aluminum liner was then installed in the opening and grouted in place using a liquid grout consisting of one part water to one part sulfaset rock bolt cement by weight. The model was tested at a principal stress ratio $\sigma_H/\sigma_V = 2/3$ to a maximum vertical model pressure of 1300 psi under plane strain conditions. The model was instrumented with eight pairs of buried extensometers and six diametrical extensometers in the tunnel liner as shown in Fig 41.



Vertical

Horizontal

Fig. 40 Photograph of JB#4 Before Test at $N = \sigma_H/\sigma_V = 2/3$ (After Hendron, et al. 1972)

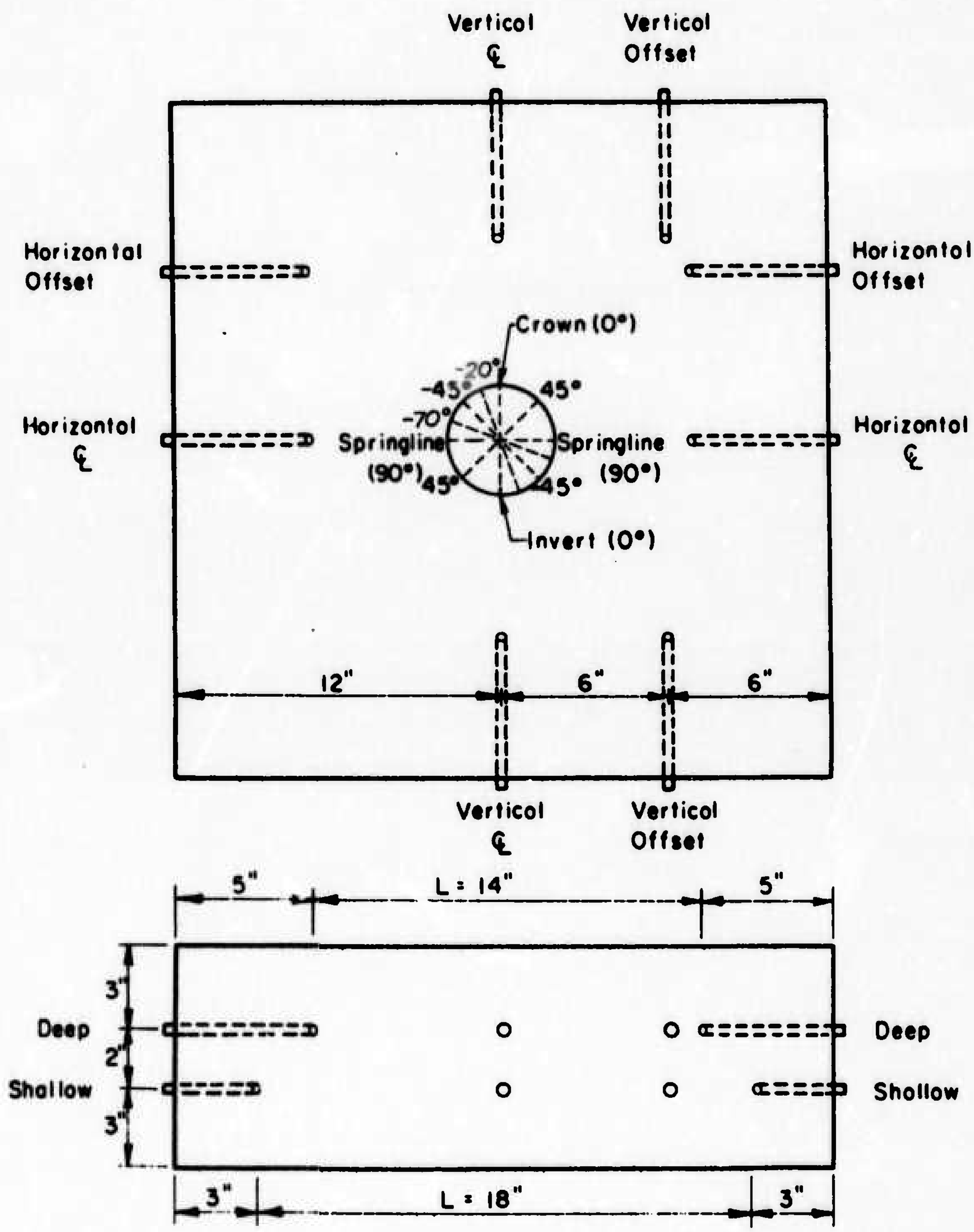


Fig. 41 Locations of Extensometers in Jointed Test Blocks (After Hendron et al., 1972)

Behavior of Model

The average stress-strain curves of the model obtained from the buried extensometers are shown in Fig. 42. The average strains represent the strains of the total model block. Both the vertical and horizontal strains are compressive. The vertacal strain is approximately three times larger than the horizontal strain. The liner buckled at a vertical pressure of about 1100 psi. The buckling of the liner is reflected in the average stress-strain curves of the block as a sharp deviation from linearity at a pressure of about 1100 psi.

The diametrical strains measured by the six diametrical extensometers are plotted in Fig. 43 as a function of the vertical model pressure. Buckling of the liner is clearly shown at a vertical pressure of about 1100 psi at which the diametrical strains increased at greater rates. A photograph of the jointed block after test is shown in Fig. 44. It may be seen that the actual buckling occurred as a pair of buckles located along the 45° diametrical plane which coincided with the intersection of a joint plane with the tunnel liner.

Analysis of Model Study

Idealization of the Model - The finite element idealization of the jointed model with a lined opening is shown in Fig. 45. The joints were idealized by one-dimensional joint elements, and the rock blocks were idealized by one or several two-dimensional elements. Because of the variation in the size of blocks and the difficulty involved in the assemblage of the model, the joint blocks of the actual model were not separated by continuous straight joints. For the idealization necessary for the analysis, the joints had to be assumed as straight and continuous.

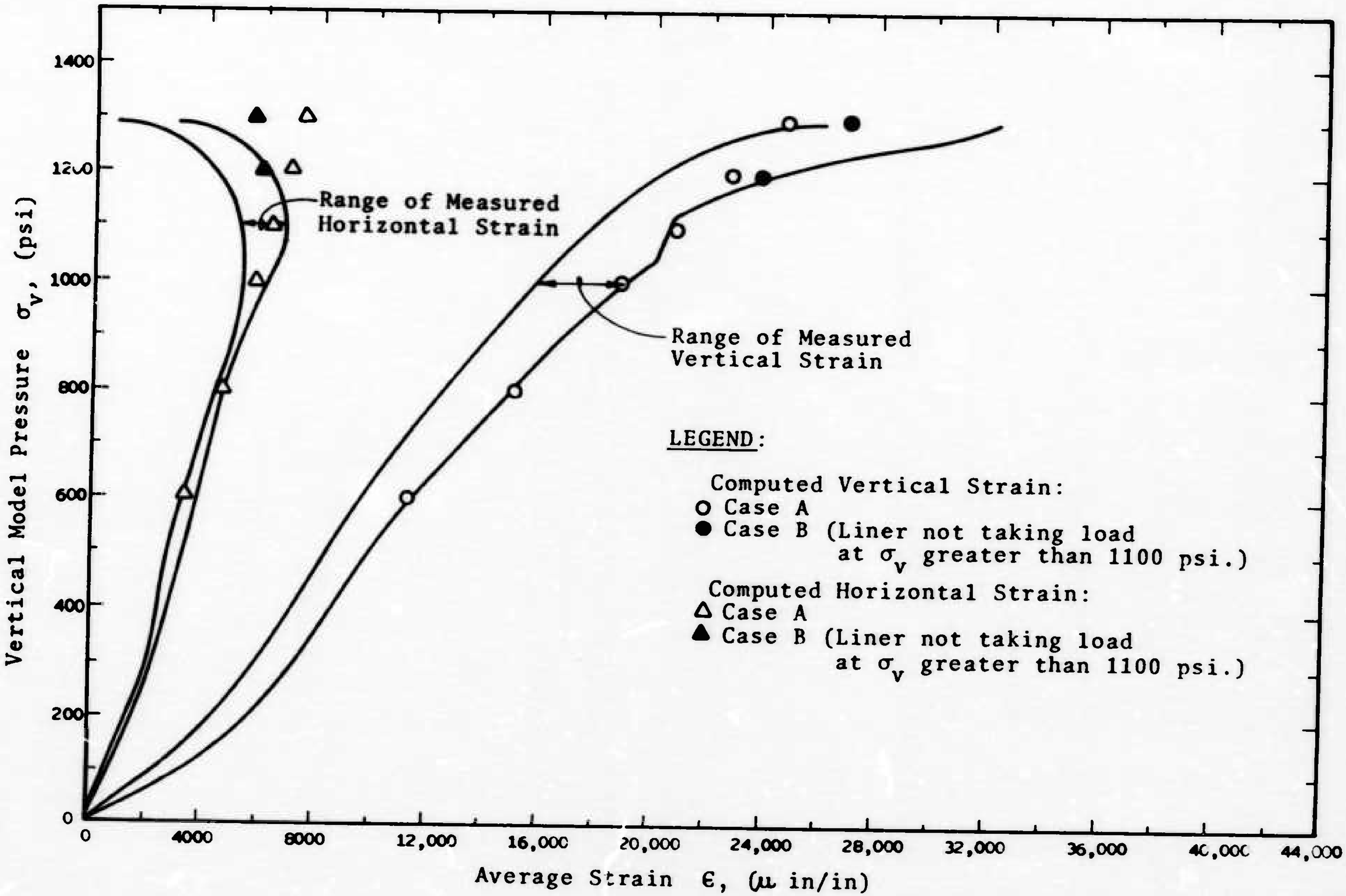


Fig. 42 Measured and Computed Average Stress-Strain Curves of Model Test on Joint Block with Lined Opening

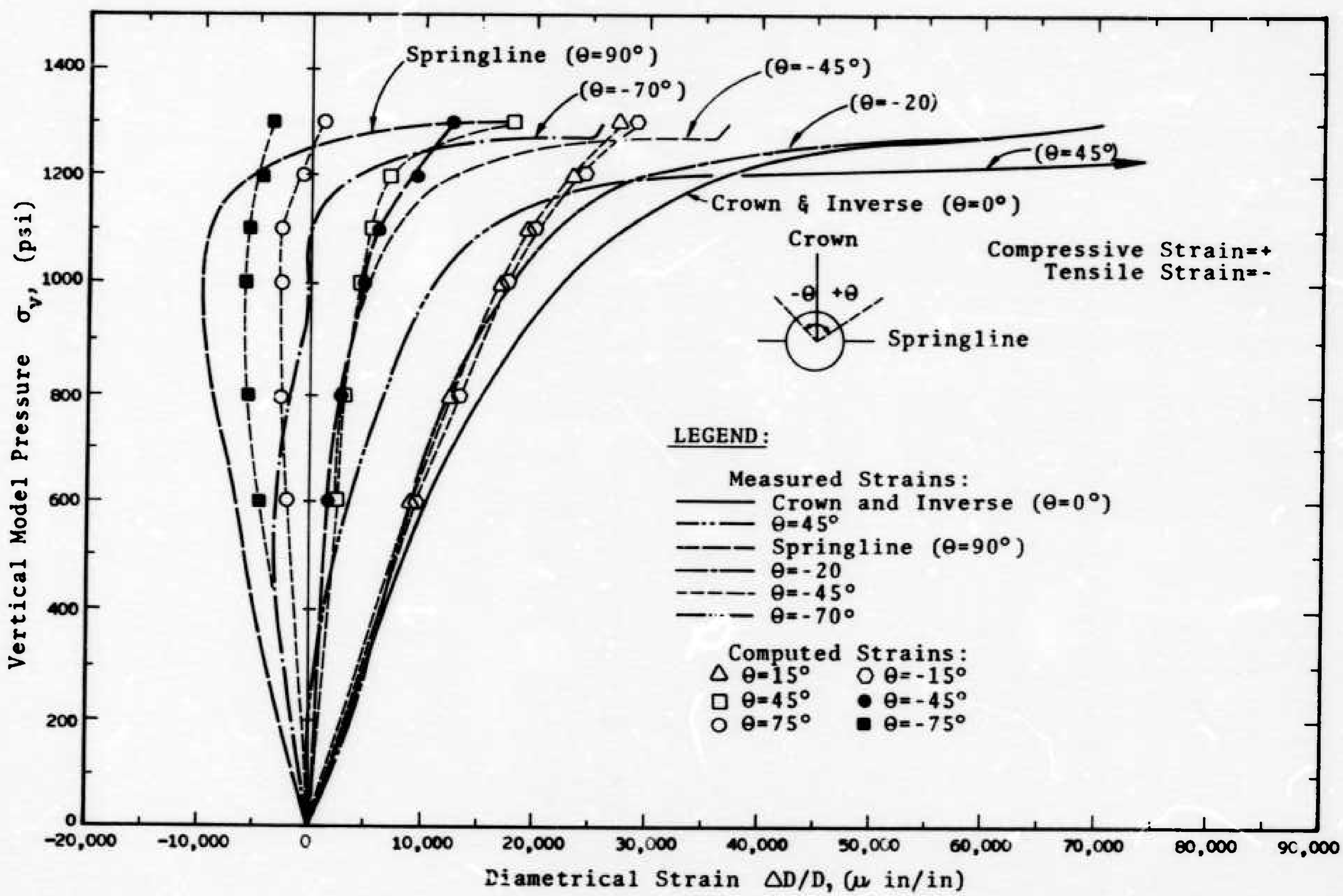
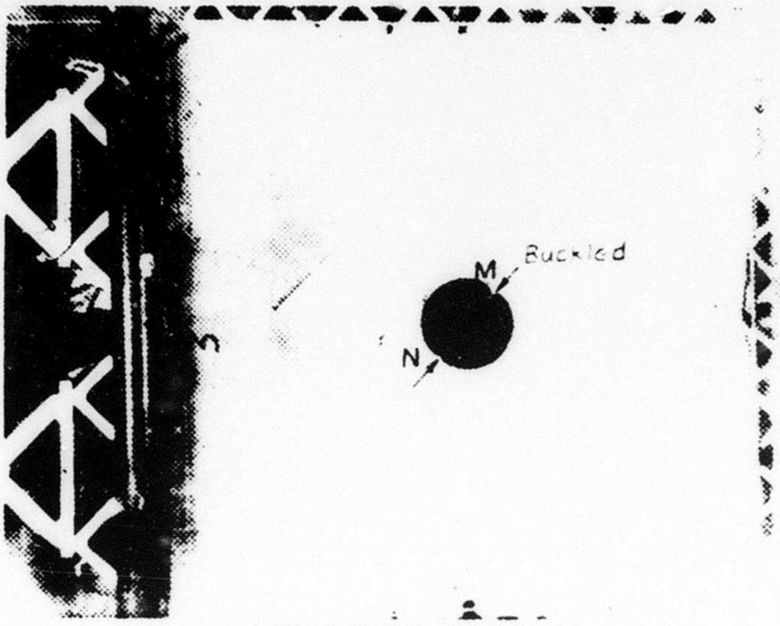
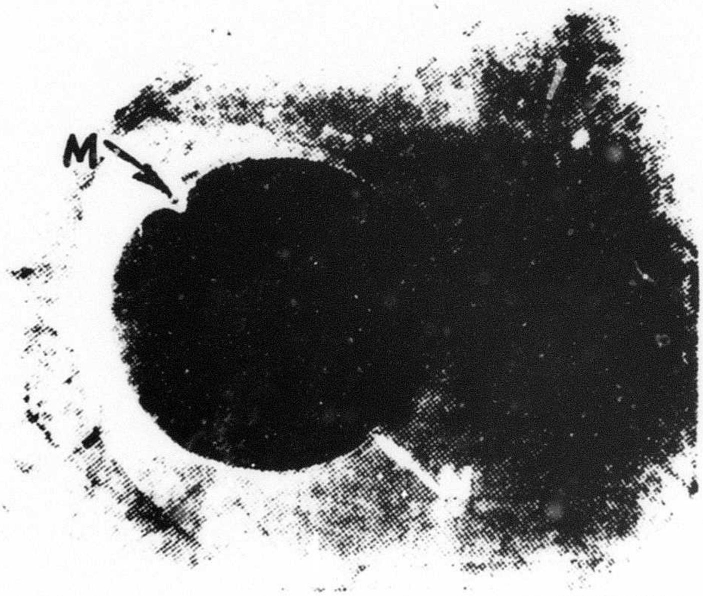


Fig. 43 Measured and Computed Diametrical Strains of Liner as a Function of Vertical Applied Pressure - Case B

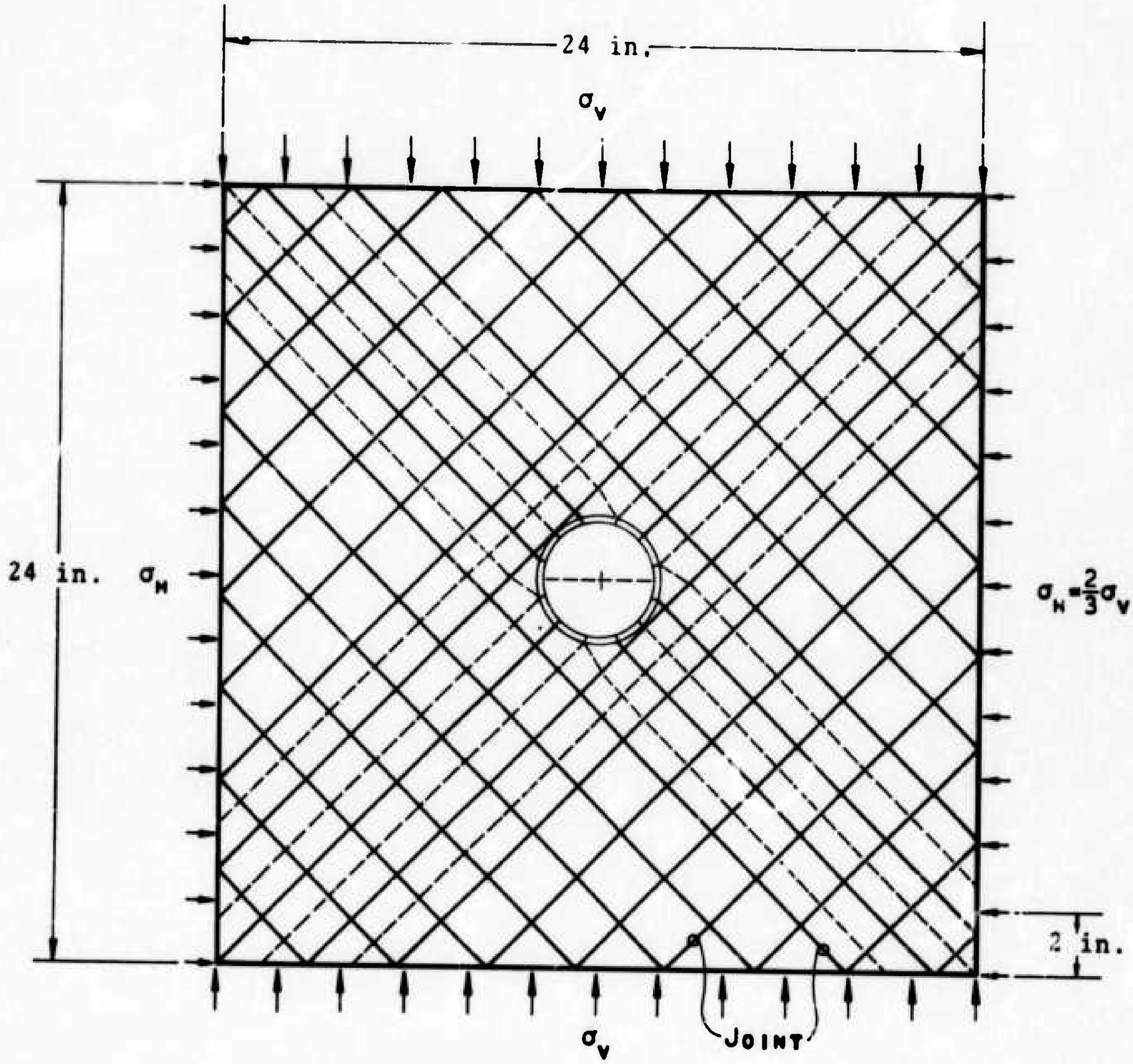


(a) View of whole model after test



(b) Closeup of tunnel liner after test
(photograph taken from the other side)

Fig. 44 Photographs of Jointed Block After Test
(After Hendron et al., 1972)



Note:
Thickness of liner
not in scale.

Fig. 45 Finite Element Idealization of Jointed Model

Material Properties Used in the Analysis - The material properties of the intact block used in the model were selected on the basis of results obtained from a series of triaxial tests conducted by Heuer and Hendron (1971), and Hendron et al. (1972). The Mohr envelope of the rock-like material used is shown in Fig. 46. The strength parameters were determined to be c = 170 psi, ϕ = 32.5 degrees, and the tensile strength of 33 psi. The average value of the modulus was determined to be 833,000 psi and the Poisson's ratio to be 0.14.

The aluminum liner was assigned the modulus of 10^7 psi and the Poisson's ratio of 0.33. The aluminum was assumed to follow the Von Mises yield criterion with the tensile and compressive strength of 40,000 psi.

As described previously, the joint blocks were sawed from larger compacted blocks. The deformability of the joints which consists of the normal and shear stiffnesses depends on the roughness of the sawed surface. No test data were available for evaluation of the deformability of the joints. The angle of shearing resistance on the joint surfaces was obtained by Hendron, et al. (1972) from a series of direct shear tests. The results of these tests are shown in Fig. 47. The effective angle of shearing resistance decreases from 33° to 29° with increasing normal pressures. The effective angle of shearing resistance of the joint surfaces used in the analysis was selected to be 29°.

Table 2 summarizes the material properties except the deformability of the joints used in the analysis of the model test.

Selection of Joint Deformation Characteristics - A parametric study was conducted assuming various combinations of both normal

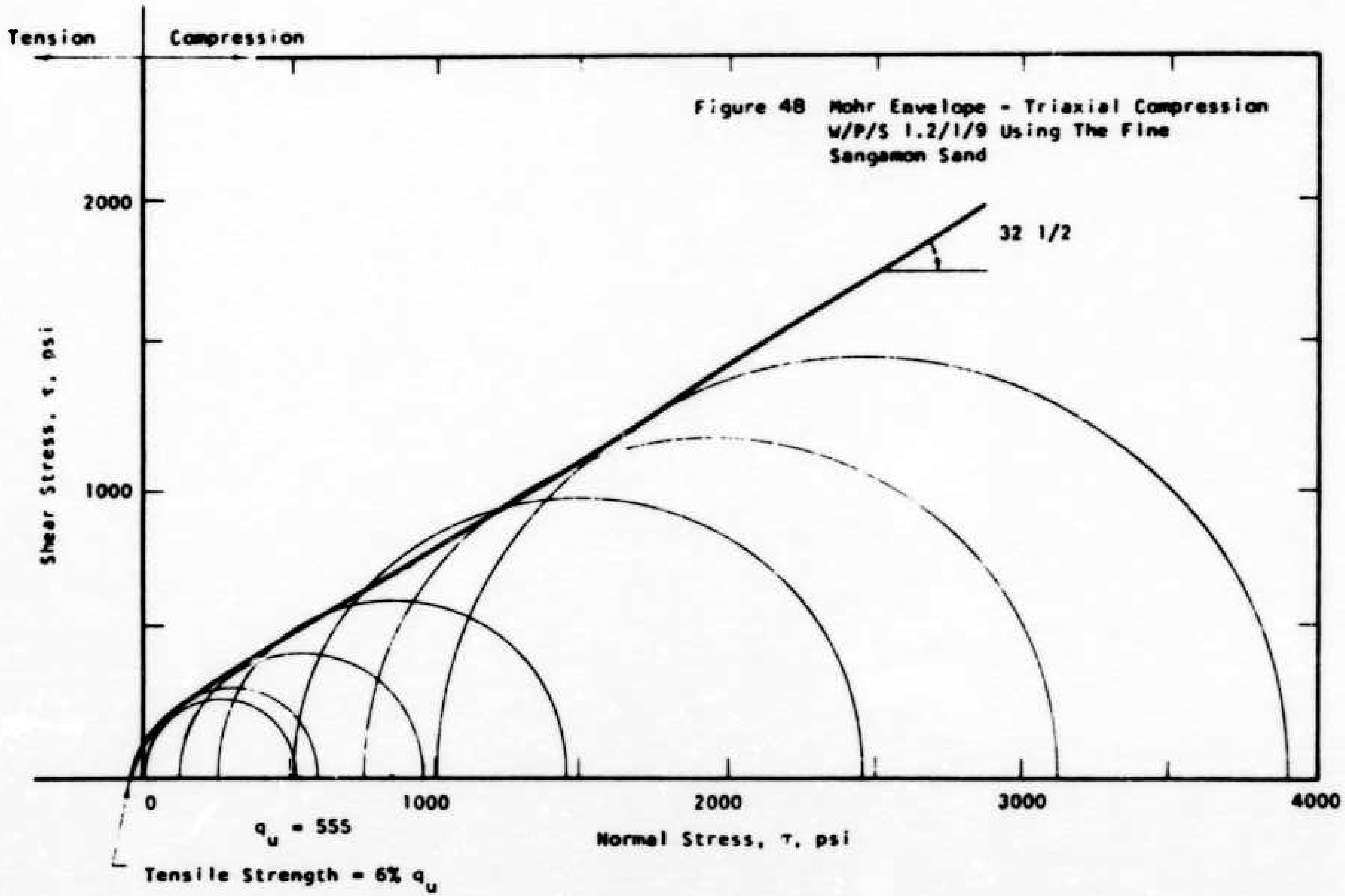


Fig. 46 Mohr Envelope for Intact Block of Rock-like Material
(After Hendron, et al. 1972)

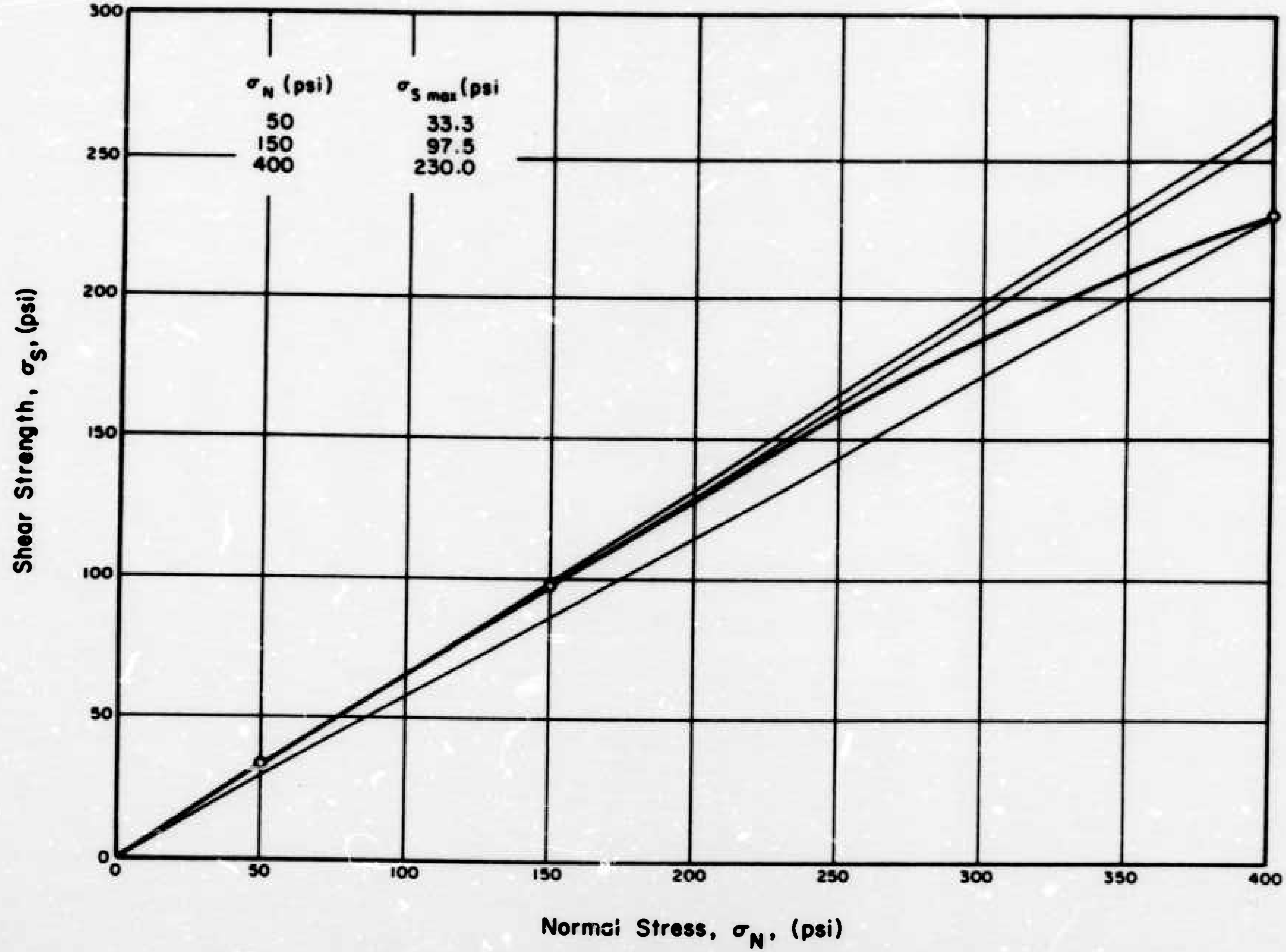


Fig. 47 Results of Direct Shear Tests on Joint Surfaces of Model Material (After Hendron et al., 1972)

Table 2. Summary of Material Properties Used in the Analyses of Model Test on Joint Block

Material	Modulus psi	Poisson's Ratio	C psi	ϕ deg.	Tensile Strength psi
Intact Block	833,000	0.14	170	32.5	33
Aluminum Liner	10,000,000	0.33	20,000	0	40,000
Joint	--	--	0	29.0	0

and shear stiffnesses in order to select a reasonable set of the joint deformability characteristics for the detailed analysis. A total of 13 cases were analyzed. The normal stiffness was varied from 2.0×10^4 pci to 4.17×10^7 pci and the shear stiffness was varied from 4.0×10^3 pci to 1.5×10^4 pci. The results of the parametric study in terms of the average model strains and the diametrical strains of the liner at the vertical model pressure of 1000 psi are summarized in Table 3.

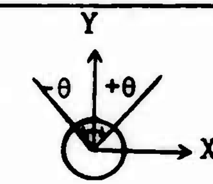
From the results of the parametric study, the deformability of the joints which would provide magnitudes of both vertical and horizontal strains similar to those observed at the applied vertical pressure of 1000 psi was selected for the detailed analysis. The normal and shear stiffnesses of the joints selected were 3.5×10^4 pci and 1.5×10^4 pci, respectively. The magnitudes of these joint stiffnesses were later found to be reasonable and compatible with the deformability of various rock joints compiled by Goodman (1969).

Results of Analysis - The following three cases were analyzed: Case A assumed that the liner continued to remain intact with the joint block at a vertical pressure greater than 1100 psi; Case B assumed that the liner buckled at the vertical pressure of 1100 psi and would not take any load at greater pressures; Case C analyzed the unlined cavity.

The results of these analyses are presented in Figs. 42, 43, 48, 49, 50 and 51. Fig. 42 shows the computed and measured average and horizontal strains as a function of the applied vertical pressure. For both Cases A and B, the computed and measured strains are in good agreement up to a vertical pressure of about 1200 psi at which point the cavity elongated in the horizontal direction due to buckling of the liner. Fig. 43

Table 3. Summary of Parametric Study of Model Test on Jointed Block

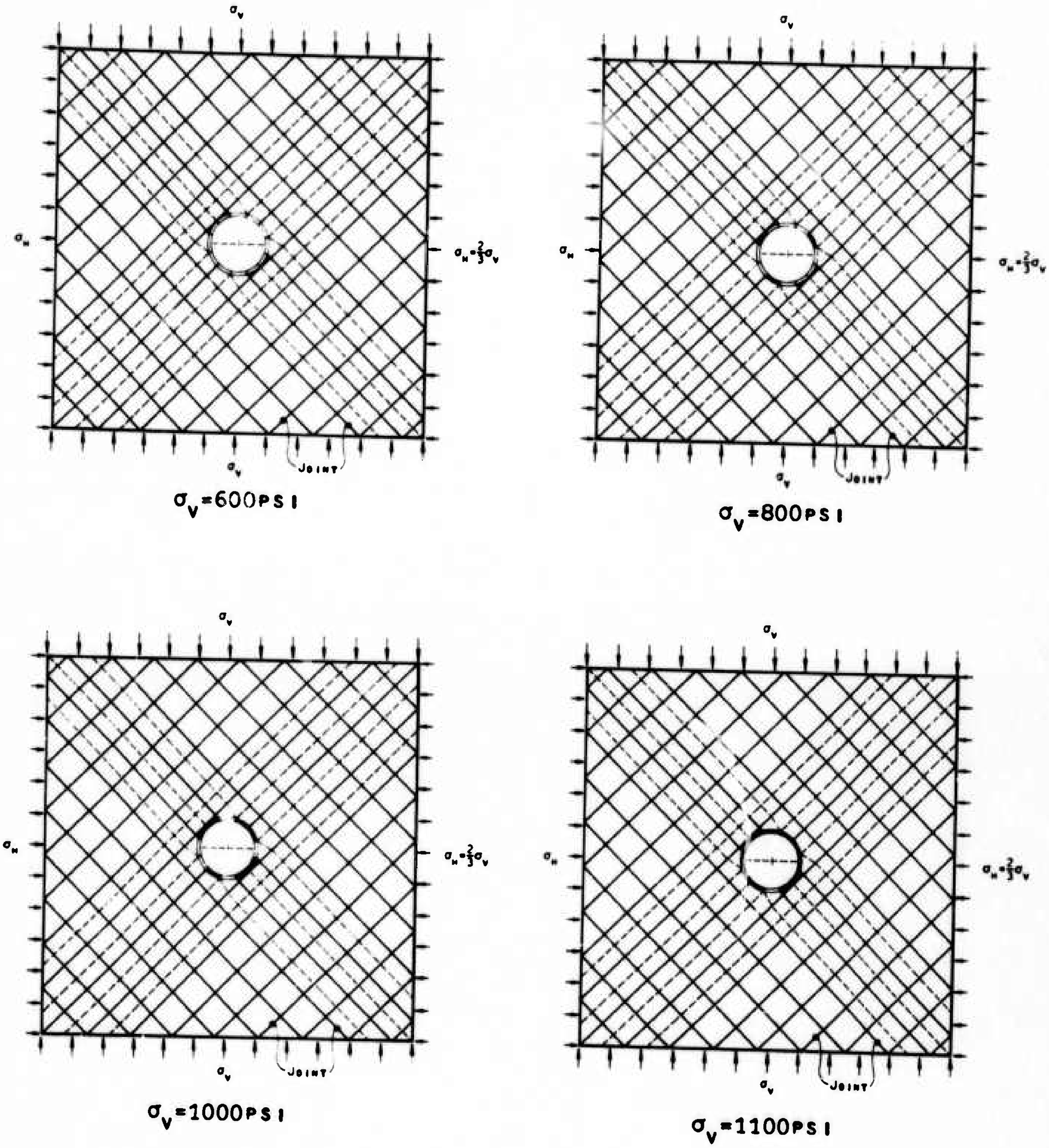
Case	K_N PCI	K_S PCI	**Average Strains μ in/in		**Diametrical Strains of Liner μ in/in			
			Vertical	Horizontal	θ = 15°	θ = 45°	θ = 75°	θ = -45°
1	4.17×10^{7}	4.0×10^{3}	19,700	-18,200	9,700	-2,100	- 5,600	3,200
2	4.17×10^{7}	8.0×10^{3}	10,700	- 9,200	7,400	- 760	- 3,300	2,900
3	4.17×10^{6}	4.0×10^{3}	19,900	-18,300	10,100	-2,160	- 5,700	3,380
4	4.17×10^{5}	4.0×10^{3}	22,000	-19,000	16,300	-4,800	-10,200	4,900
5	4.17×10^{5}	5.0×10^{3}	18,100	-15,000	14,000	-3,050	- 8,100	4,400
6	8.0×10^{4}	5.0×10^{3}	22,800	-10,800	13,800	570	- 7,600	3,600
7	4.17×10^{4}	5.0×10^{3}	27,900	- 6,600	19,800	3,700	- 9,500	2,700
8	3.0×10^{4}	1.0×10^{4}	22,900	4,900	17,700	4,400	- 5,900	2,200
9	3.0×10^{4}	1.3×10^{4}	20,900	5,200	16,400	4,600	- 4,600	2,250
10	2.5×10^{4}	1.3×10^{4}	23,500	9,200	17,700	5,300	- 5,000	1,900
11	2.0×10^{4}	1.2×10^{4}	28,000	12,400	19,900	6,300	- 5,900	1,300
12	3.0×10^{4}	1.5×10^{4}	20,100	7,550	15,800	4,650	- 4,000	2,290
***13	3.5×10^{4}	1.5×10^{4}	18,200	5,810	14,800	4,090	- 3,750	2,540
*Measured Strains $\sigma_v = 1{,}000$ psi, $\sigma_h = \frac{2}{3}\sigma_v$			17,880	5,940	θ = 0° 22,000	θ = 45° 11,000	θ = 70° 0	θ = -45° 5,000
					θ = 20° 18,000		θ = 90° -10,000	



$\sigma_v = 1{,}000$ psi

$\sigma_h = \frac{2}{3}\sigma_v$

*After Hendron, et al. 1972
**Compression = +
Extension = -
***Joint stiffness used in the detailed analysis



LEGEND:

▬ Elements Yielded

Fig. 48 Development of Yielded Zones in Liner

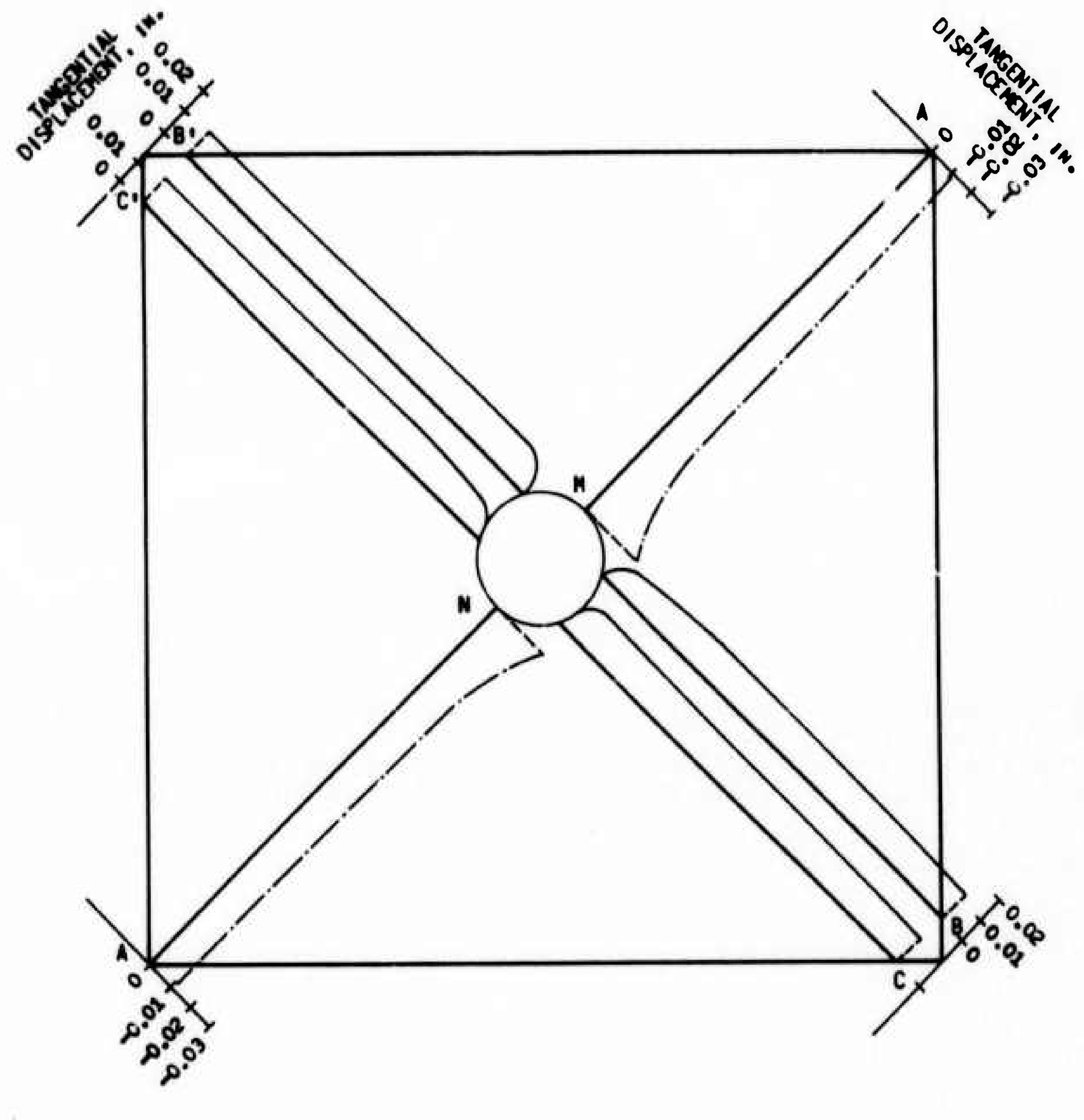


Fig. 49 Relative Tangential Displacements Across Joints in the Vicinity of Opening at Vertical Stress of 1100 psi

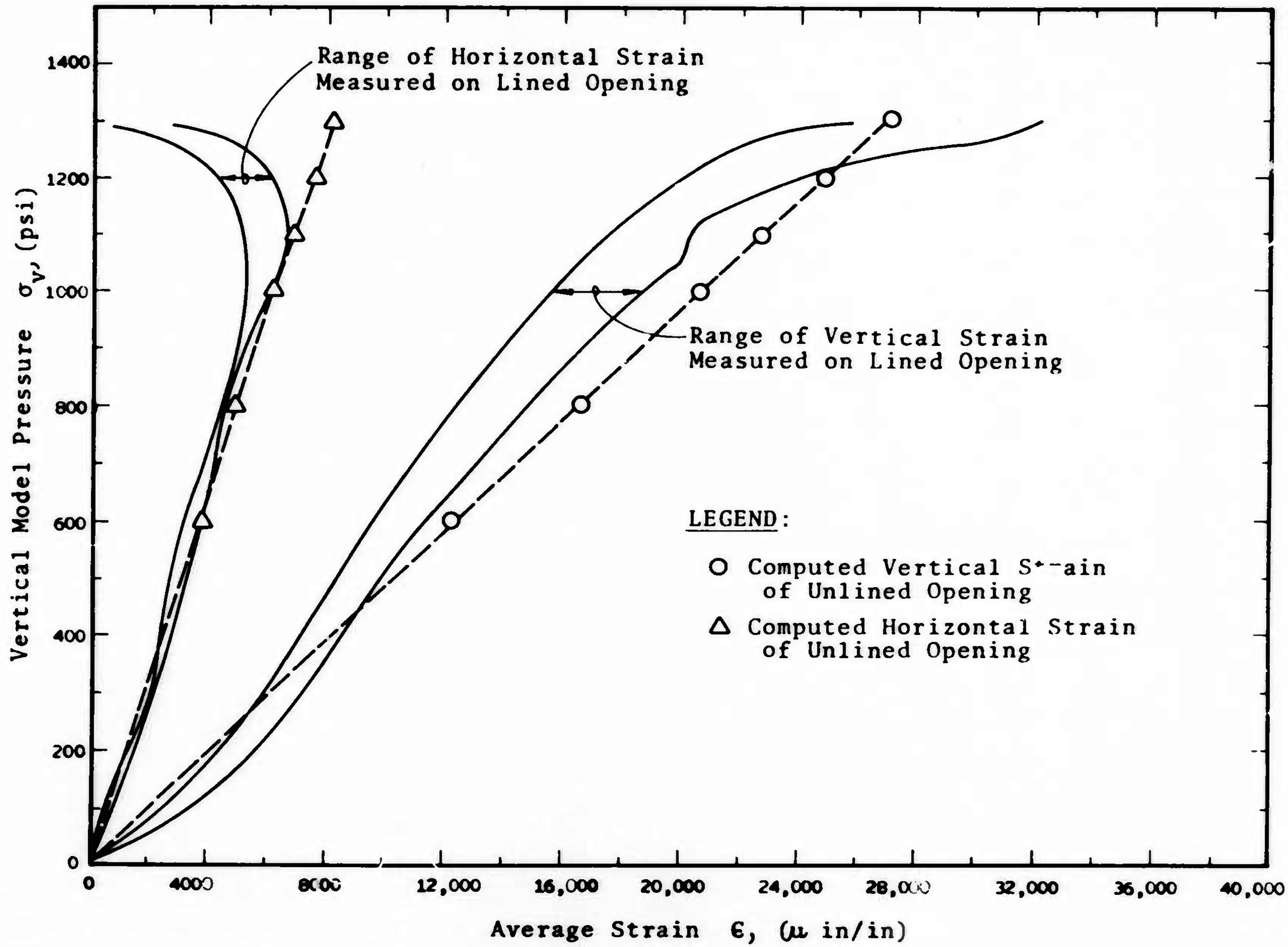


Fig. 50 Computed Average Stress-Strain Curves of a Jointed Model With an Unlined Opening

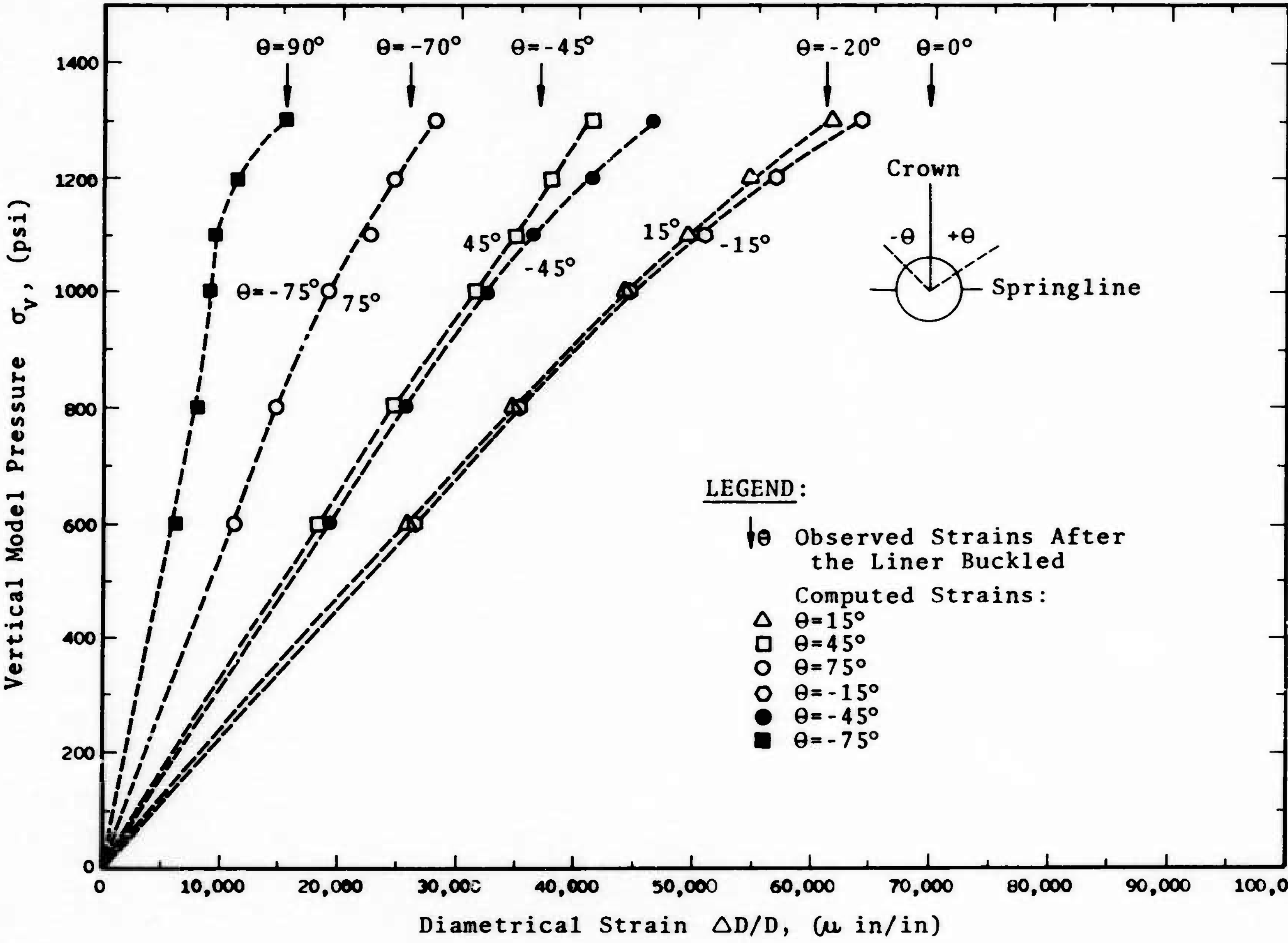


Fig. 51 Computed Diametrical Strains of an Unlined Opening of a Jointed Model

shows the computed diametrical strains obtained from the analysis of Case B together with the measured strains at various locations along the liner. Generally, the magnitudes of the computed strains are in reasonable agreement with those measured at their respective locations along the liner. However, the rate of increase in the measured diametrical strain is greater than those computed at vertical pressures greater than 1100 psi.

Fig. 48 illustrates the propagation of computed plastic zones in the model as well as the liner with an increase in the applied pressure. The plastic zones are confined to the liner and the immediate vicinity of the opening. It is interesting to note that at the vertical pressure of 1100 psi, the plastic zones appear to propagate over the entire liner, a result consistent with the observed buckling of the liner at the same applied pressure.

To examine the cause of buckling of the liner which occurred along the 45° diametrical plane (MN) as shown in Fig. 44, relative tangential displacements across the joints in the vicinity of the opening at the applied vertical pressure of 1100 psi are plotted in Fig. 49. It may be noted that the relative tangential displacements across the joint along the 45° diametrical plane AA' increased from 0.01 inch at the locations away from the opening to 0.02 inch at the intersection of the joint and the liner, indicating that the joint plane was penetrating the liner at the location where the buckling of the liner occurred.

The results of the analysis on the unlined opening (Case C) are shown in Figs. 50 and 51. The comparison between the average model strains of the lined and unlined openings indicates that the liner has an insignificant effect on the average strains of the models. However, the comparison between the diametrical strains computed for the unlined opening shown in Fig. 51 and those for the lined opening shown in Fig. 43, indicates that

the liner has a large effect in restraining the inward movements of the opening. Also shown in Fig. 51 are the diametrical strains measured after the liner buckled in the model test described in the previous section. It may be noted that at the vertical pressure of 1300 psi, the computed diametrical strains are on the same order of magnitude as those measured after the liner buckled, indicating that the behavior of the opening after the liner failed could be predicted by analyzing the opening with no liner.

Summary

The results of the analyses of the laboratory model of the lined opening in jointed rock conducted by Hendron et al. (1972) indicate that the behavior of the model is greatly affected by the deformability of the joints expressed in terms of the normal and shear stiffnesses. Although no data were available to determine the deformability of the joints, it was possible through an iterative process to determine joint stiffnesses which would provide a reasonable agreement between the computed and measured deformabilities of the model. Using these joint stiffnesses, the behavior of the lined opening up to the buckling of the liner could be predicted with reasonable accuracy. The discrepancy between the computed and measured diametrical strains may be attributed to the approximations involved in the idealization of the actual jointed model and the interaction of the liner and the jointed blocks in the vicinity of the opening.

A comparison of the results of the analyses of the lined and unlined openings showed that the liner had a significant influence in restraining the inward movement of the opening. It was also found that the behavior of the lined opening after the failure of the liner could be predicted from an analysis of the unlined opening.

ANALYSIS OF TUMUT I UNDERGROUND POWER STATION, SNOWY MOUNTAINS, AUSTRALIA

Description and Geological Structure of the Site

The Tumut I underground power station (Moye, 1964) is situated under the lower part of the very steep eastern wall of the Tumut Valley in the Snowy Mountains of southeast Australia, about 1100 feet vertically below the ground surface, 1200 feet in from the river, and 150 feet below the level of the river bed. The plan of the power station is shown in Fig. 52. The machine hall is 306 feet in length, 44 feet in maximum width, and 104 feet in maximum height.

The Tumut I power station is located in a complex mass of granitic parogneiss and granulite intruded by sheets of granites. The group of metamorphic rocks is referred to as Boomerang Creek granitic gneiss, and the granites as Happy Valley granite. The granite sheets strike N65°E to N100°E and dip 40°-50° SE. Their distribution at power station level is shown in Fig. 52.

At Tumut I power station site, two small but persistent minor faults intersect the machine hall. One is over the full length of the roof (Fig. 52, A). It strikes N40°E to N60°E and dips approximately 35°SE. In the granite, it is seen as one or a group of several persistent fracture planes with 1/2 to 1 inch of crushed granite containing a little clay along the planes, or as a zone of close jointing. In the gneiss, it is represented by a zone of close jointing 5-10 feet wide, with joints spaced 2-6 inches apart. These joints are usually smooth, coated with chlorite but <u>not clay</u>, slickensided, and tightly closed. The second small fault has a strike of N30°E and dips 60°-70°W. It cuts across the tailrace surge chamber, draft tubes, and the western end of the machine hall (Fig. 52, B). In the granite it usually consists of one or two fracture planes with 1/4 to 1 inch of <u>clay</u> and crushed granite along the planes. As the contact with the gneiss is approached, it

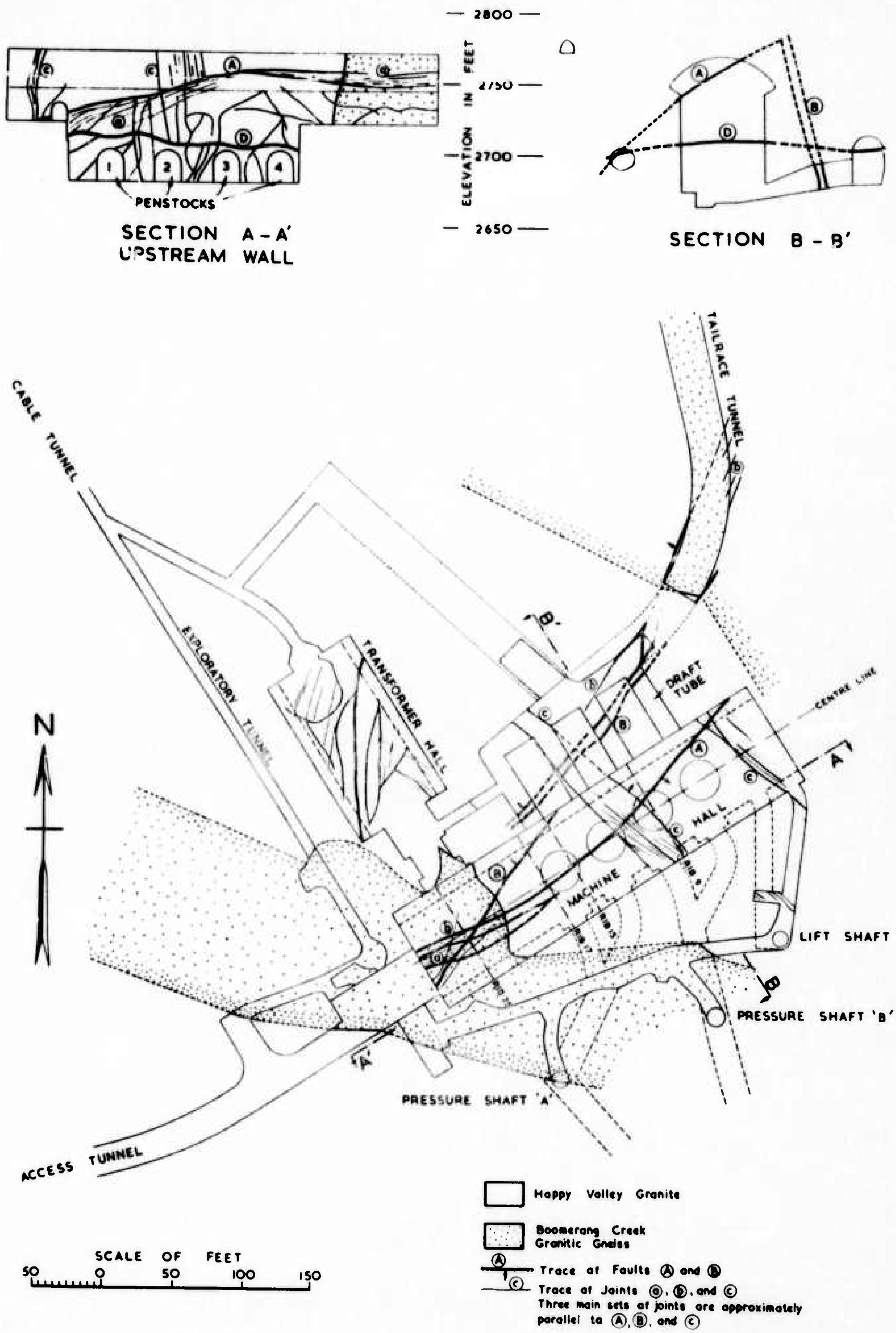


Fig. 52 Plan and Sections of the Tumut I Power Station

becomes less distinct and splits into several parallel clay-coated joints, many without crushed rock, and continues in the gneiss as a group of clay-coated joints.

In addition to these minor faults, there is a very persistent zone of close fracturing a few inches in width, which occurs in the lower part of the power station walls, and in the walls of the transformer hall (Fig. 52, D). This zone has been found only in the granite. It has a roughly north-south strike and a gentle dip to the east.

In addition to these well-defined localized structures the rock mass is extensively jointed. The joint pattern is similar in both rock types, but the spacing of the joints is usually much closer in the gneiss than in the granite. Most joints can be grouped into the following three principal sets (Fig. 52, a, b, c):

Set a: Strike N40°-60°E and dip 35°SE. These are parallel to fault A.

Set b: Strike N30°E and dip 65°W to 80°E. The strike of these joints makes a small acute angle with the long walls of the machine hall.

Set c: Strike N130°E and dip 80°W. These joints are spaced 40-80 feet apart but are very persistent. A single joint may split into two or more closely spaced joints. The joint surfaces are rough and irregular. The strike of these joints is nearly at right angles to the long walls of the machine hall.

In the gneiss the spacing of the joints of sets a and b is generally 6 inches to 2 feet with some areas of narrower and some of wider spacing. The joint surfaces are usually smooth

and slickensided. Most joints are tightly closed. In the granite, the spacing of the joints of sets a and b is variable but generally in the range of 1 to 5 feet.

Construction Sequences and Behavior of the Excavations

The machine hall was excavated in several stages. The detail of excavation sequences is shown in Fig. 53. After the pilot tunnel was driven, the roof section of the machine hall was excavated to full width, rock bolts and permanent concrete ribs installed. Following this, the main body of the machine hall was excavated by quarrying methods. The vertical walls and roof were systematically rock-bolted as soon as they were exposed (Moye, 1964; Lang, 1958). The rock bolts used consisted of mild steel bars 1 inch in diameter, mostly 10 or 15 feet long, with a slot-and-wedge-type anchor and furnished with 6-inch- or 8-inch-square steel plates for bearing against the rock surface. They were spaced 4 or 5 feet apart. During installation, they were stressed to a nominal load of 20,000 pounds tension. The concrete ribs were of 4 feet x 4 feet cross-section, with a spacing between ribs of 8 feet over one-half of the hall and a spacing of 4 feet over the other half, where the rock was more jointed. During excavation, overbreak to the extent of 1 to 3 feet was common under the effects of blasting. On the upstream wall rather extensive loosening of the granite joint blocks occurred. This loosening apparently was influenced by the zone of close jointing along fault A in the middle of the wall being intersected by joints of set b dipping steeply toward the excavation.

The behavior of the rock mass around the machine hall excavation was observed during construction by the following quantitative instrumental measurements:

(a) The strain in many of the reinforced concrete arch ribs was measured by means of electric resistance-type

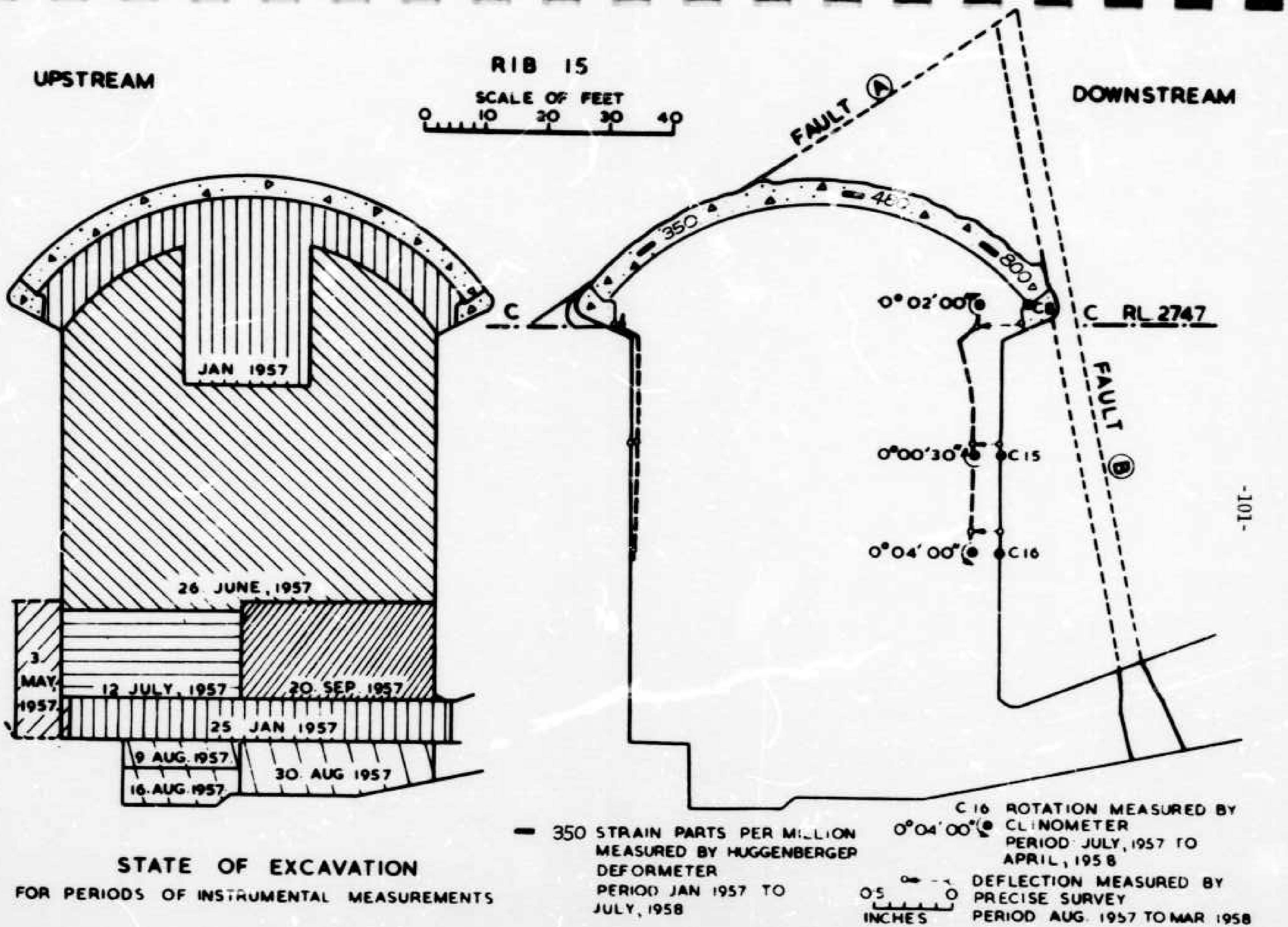


Fig. 53 Construction Sequences and Movements in the Machine Hall of Tumut I Underground Power Station (After Alexander et al. 1963)

strain meters embedded in the concrete, and by observation of Huggenberger deformeter points fixed on the surface of some ribs.

(b) The horizontal movement of points at the ends of the concrete ribs and on the rock walls was measured by precise survey methods.

(c) The angular rotation of points on the reinforced concrete abutment beams and on the walls was measured by means of sensitive clinometers.

The data obtained from the instrumental measurements were summarized by Alexander, et al. (1963) and shown in Fig. 54. At Rib No. 15, the strains developed in the roof rib one year after the excavation was complete were 800 μin./in. at the downstream side. The abutment deflections at RL 2746 developed in the 5-month period after the excavation was completed were 0.3 inch at the downstream side and 0.07 inch at the upstream side. Both sides of the abutment moved towards the center of the hall. It may be noted that the initial measurements of the reference points were made <u>after</u> the excavation was complete. Therefore, the movements which occurred during excavation were not recorded. While the instrumental data presented in Fig. 54 were not sufficient to give a complete picture of the behavior of the excavation, certain trends are recognizable. Significant features of the behavior of the excavation are summarized below:

(1) The movements due to excavation were much larger on the downstream side of the machine hall. This behavior was manifested by large strains and deflections measured on the downstream side of the concrete ribs and the rock walls as compared with small movements observed on the upstream side. The asymmetrical behavior of the

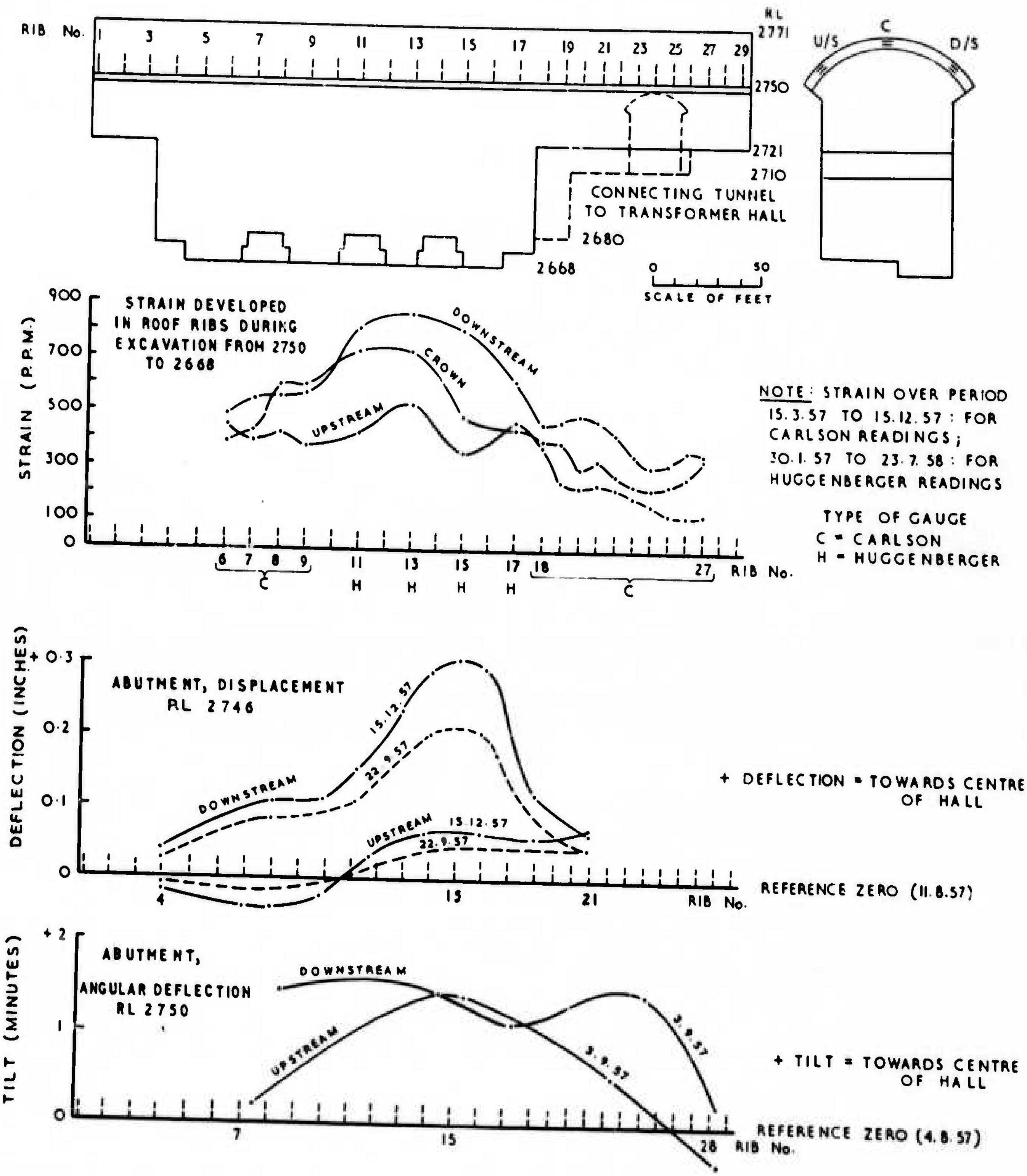


Fig. 54 Tumut I Power Stations--Observations on Ribs and Abutments (After Alexander et al. 1963)

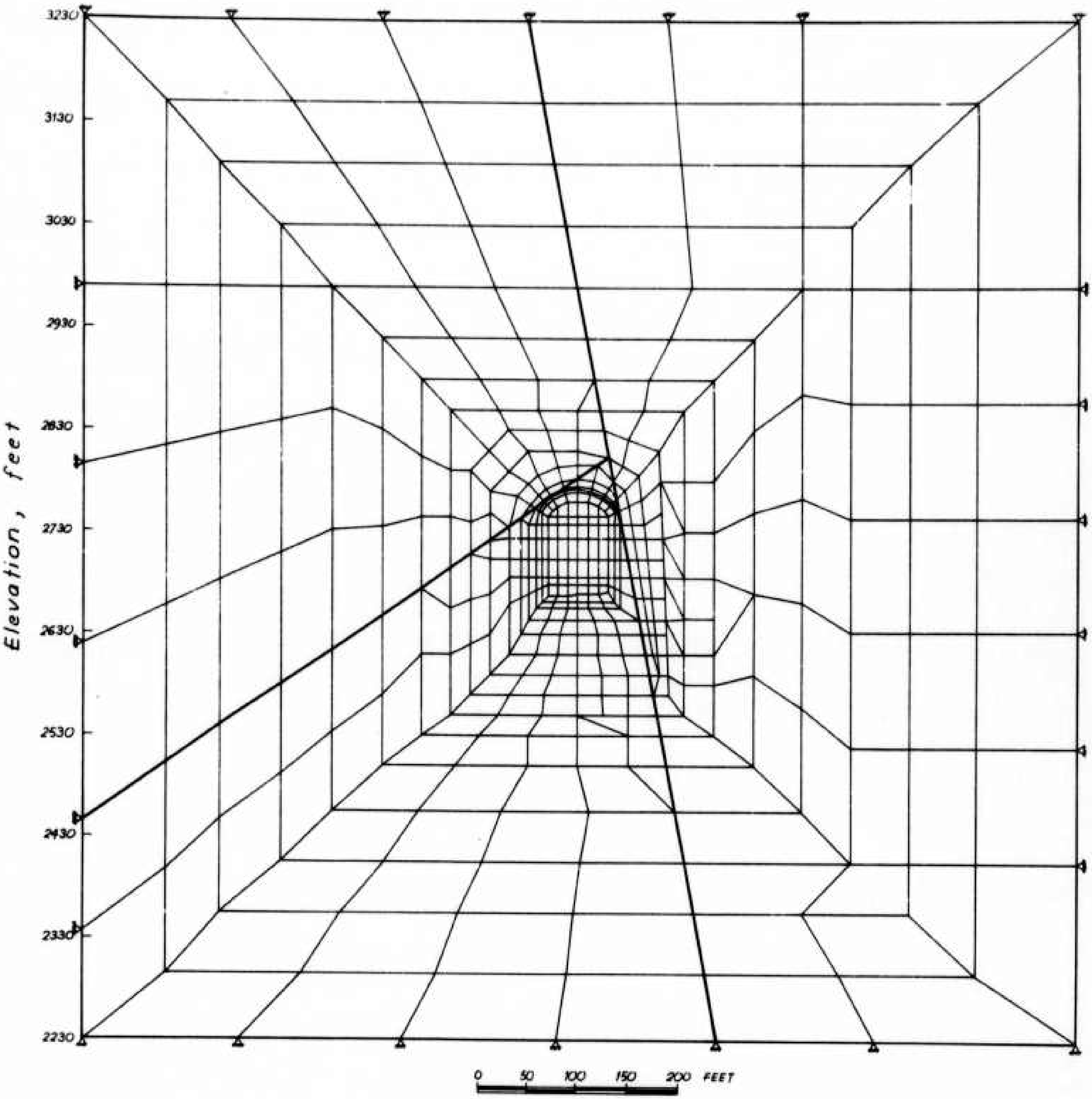


Fig. 55 Finite Element Idealization of Tumut I Power Station

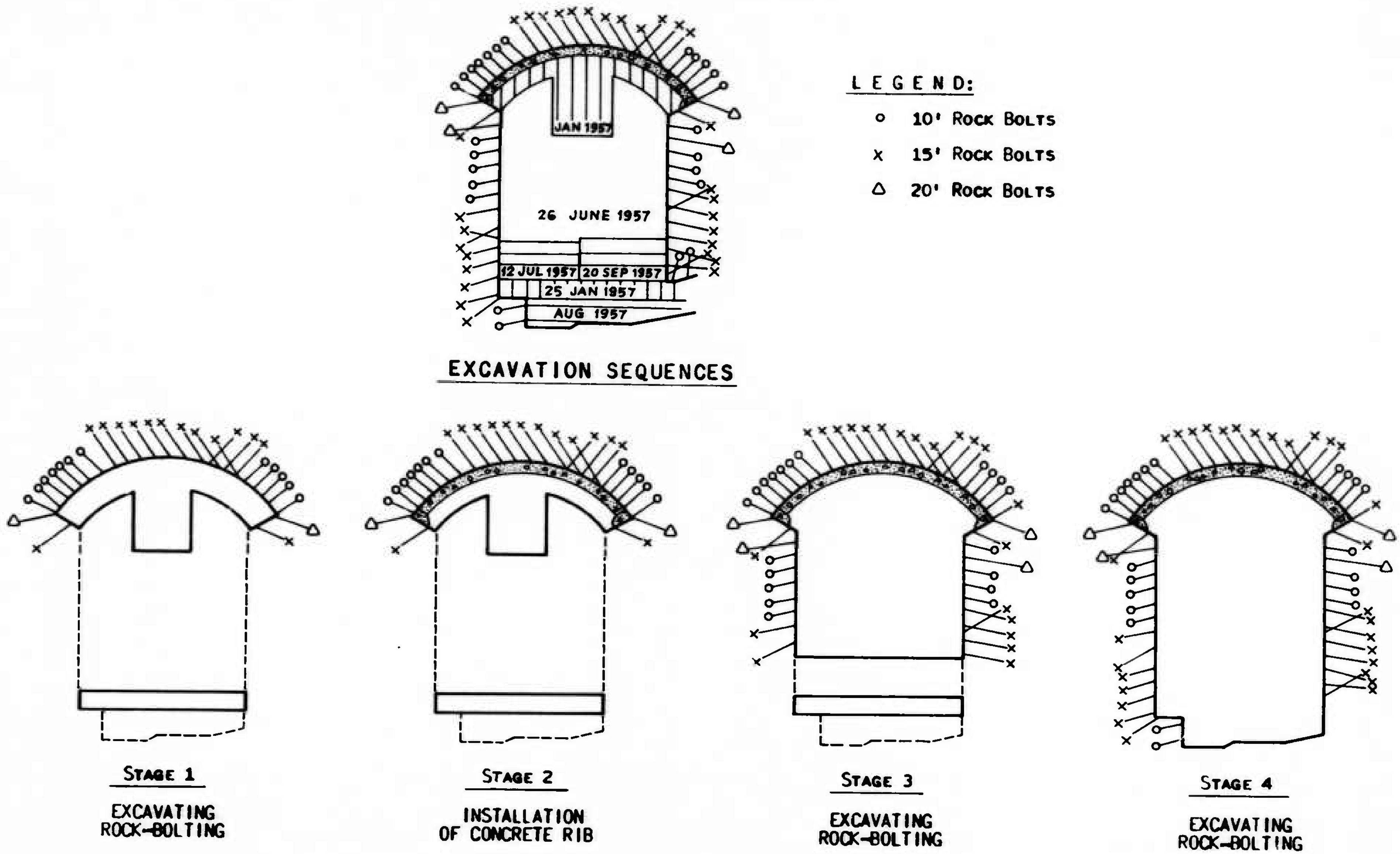


Fig. 56 Analytical Simulation of Construction Sequences for the Machine Hall of Tumut I Power Station

machine hall may be attributed to topographic effects and the existance of the two intersecting minor faults and joints near the downstream end.

(2) Severe cracking was observed in the concrete abutment beams on which the roof ribs were supported and spalling noted in a number of ribs in locations adjacent to both upstream and downstream abutments, the spalling being more severe at the downstream end. It is interesting to note that the cracking was first noticed when the excavation was nearing completion and grew progressively worse until the excavation was complete, after which no further worsening was noticed. This appears to indicate that the construction sequence had definite effects on the behavior of the support and excavation.

Idealization of the Power Station Excavation

Finite Element Idealization - For the purpose of analysis, a cross-section through the roof Rib No. 15 shown in Fig. 54 was selected. A finite element idealization of the section is shown in Fig. 55. Because the power station is situated at a depth greater than 1100 feet below the ground surface, it can be assumed that the presence of the ground surface has a negligible effect on the behavior of the excavation. For this reason, the boundaries of the finite element mesh were assumed fixed against any movements. The essential feature in the idealization is the presence of two faults, A and B. These faults are sub-parallel to the axis of the excavation. Therefore, the plane strain assumption of these geologic features should not incur serious errors in the analysis. The faults were idealized with Goodman's one-dimensional joints. To simulate actual excavation sequences (shown in Fig. 53) a simplified 4-stage excavation as shown in Fig. 56 was employed in the analysis. The concrete rib of 4 feet x 4 feet cross-section with spacing between ribs of 8 feet was idealized as a 2.8-foot-

thick continuous concrete arch with the same section modulus. The effect of patterned rock-bolting in the roof and vertical walls was simulated by applying an equivalent pressure of 7 psi to the excavated face and the interior of the rock mass along rock anchors. The pressure was applied when the excavation was made.

Initial State of Stress - Stress measurements were made at one of the machine halls of Tumut I when the power station excavation was well advanced (Alexander, et al. 1963). The measured stresses were corrected by the stress-concentration factors estimated at the test sites. The initial state of stress at the springline of Tumut I was computed to be:

vertical stress σ_v = 1800 psi
horizontal stress σ_h = 1500 psi
shear stress τ_{vh} = 250 psi

Because Tumut I is situated below a steeply sloping wall of the Tumut Valley, it was considered reasonable to assume that shear stresses and high horizontal stress would exist at the site.

In the analysis of Tumut I power station, the initial state of stress in the rock mass was computed in accordance with the following procedure: (1) the vertical stress in the rock mass was calculated by applying correction due to gravity to the assumed vertical stress of 1800 psi at the springline; (2) the horizontal stress was computed by multiplying the vertical stress by 0.83, the ratio of the horizontal to vertical stress at the springline; and (3) the shear stress was assumed constant throughout the rock mass.

Material Properties - The rock present at the section of interest is predominantly Happy Valley Granite. The strength and elastic

properties of the rock were measured from a series of triaxial tests on pieces of drill cores free from visible joints or other defects and are summarized in Table 4. In general, because of the presence of joints and fractures, the moduli of a rock mass are much less than those determined on small intact rock specimens. For the purpose of analysis, a lower value of modulus, as summarized in Table 4, was used.

Both faults A and B were idealized by one-dimensional joint elements. The properties of these faults were approximated by the normal and shear joint stiffnesses (Goodman, 1969) which are functions of normal and tangential deformability of the faults and the thickness of the fractured zones. No data was available for evaluation of the normal and shear stiffnesses. However, fault "B" appears to be more deformable than "A" because of the presence of clay and clay-coated joints in "B". A number of sets of joint stiffness was utilized to parametrically study the influence of the deformability of the faults on the behavior of the excavation. The values which provided a good agreement between the observed and computed strains in the concrete rib are summarized in Table 4.

Analysis Procedures

To study the behavior of the machine hall excavation, a four-stage construction and excavation sequence as shown in Fig. 56 was simulated in the analysis. The initial state of stress before excavation was first calculated for each element. Nodal forces to simulate excavation and rock-bolting, if any, were computed, and a linear elastic analysis was conduced at each excavation stage. Additional iterative analyses were performed to redistribute any excess stresses if two-dimensional elements yielded or failed in tension, or joint elements failed in shear or tension. The stress, strain, and deformation components due to each stage of excavation were cumulated in accordance

Table 4 Rock Properties at Tumut 1

Underground Power Station

A. Happy Valley Granite (Moye, 1964)

	Values Obtained from Tests	Values Used in Analysis
Young's Modulus, psi	6 to 10 x 10^6	3 x 10^6
Poisson's Ratio	0.16 to 0.21	0.18
C**, psi	4,000	4,000
ϕ**, deg.	44	44
Tensile Strength, psi	500 to 1,500	500
γ pcf	165	165

B. Faults "A" and "B" (Assumed)

Fault	Normal Stiffness pcf	Shear Stiffness pcf	C psi	ϕ degree
A	1 x 10^7	5 x 10^6	0	35
B	1 x 10^6	5 x 10^5	0	27

with the appropriate sequences. In this case study, the concrete ribs were installed after roof excavation, and only the subsequent excavations caused stresses and strains to develop in the roof ribs. Following the actual construction sequence, the sequential stress and strain developed in the rock mass and supports could automatically be accounted for.

Presentation and Discussion of Results

The results of the analysis presented in Fig. 57 show the distribution of maximum compressive strain computed in the concrete rib. It may be noted that considerably larger strains developed at the downstream side as compared to the strains at the upstream side. Also shown in Fig. 57 are the measured strains at the crown, downstream and upstream sides. The analysis also indicated that a tensile "cracked" zone developed at the upstream edge of the concrete rib. The comparison indicates that the observed and computed strains in the concrete rib are in fairly good agreement.

Also shown in Fig. 57 are the computed lateral wall deflections. A maximum deflection of 1.7 inches was calculated near the upper portion of the downstream wall, and 0.5 inches on the upstream wall. No data of observed wall deflections were available for comparison. However, the observed wall deflections developed after the excavation was essentially completed indicate that movements on the downstream wall were greater than those on the upstream wall, a result similar to that obtained from the computations. The larger strains in the concrete rib and the larger wall deflection at the downstream side than those at the upstream side indicate that the presence of the fault "B" had a significant effect on the behavior of the excavation.

Evaluation of the stress distribution in the rock mass surrounding the excavation showed that the stress levels were well

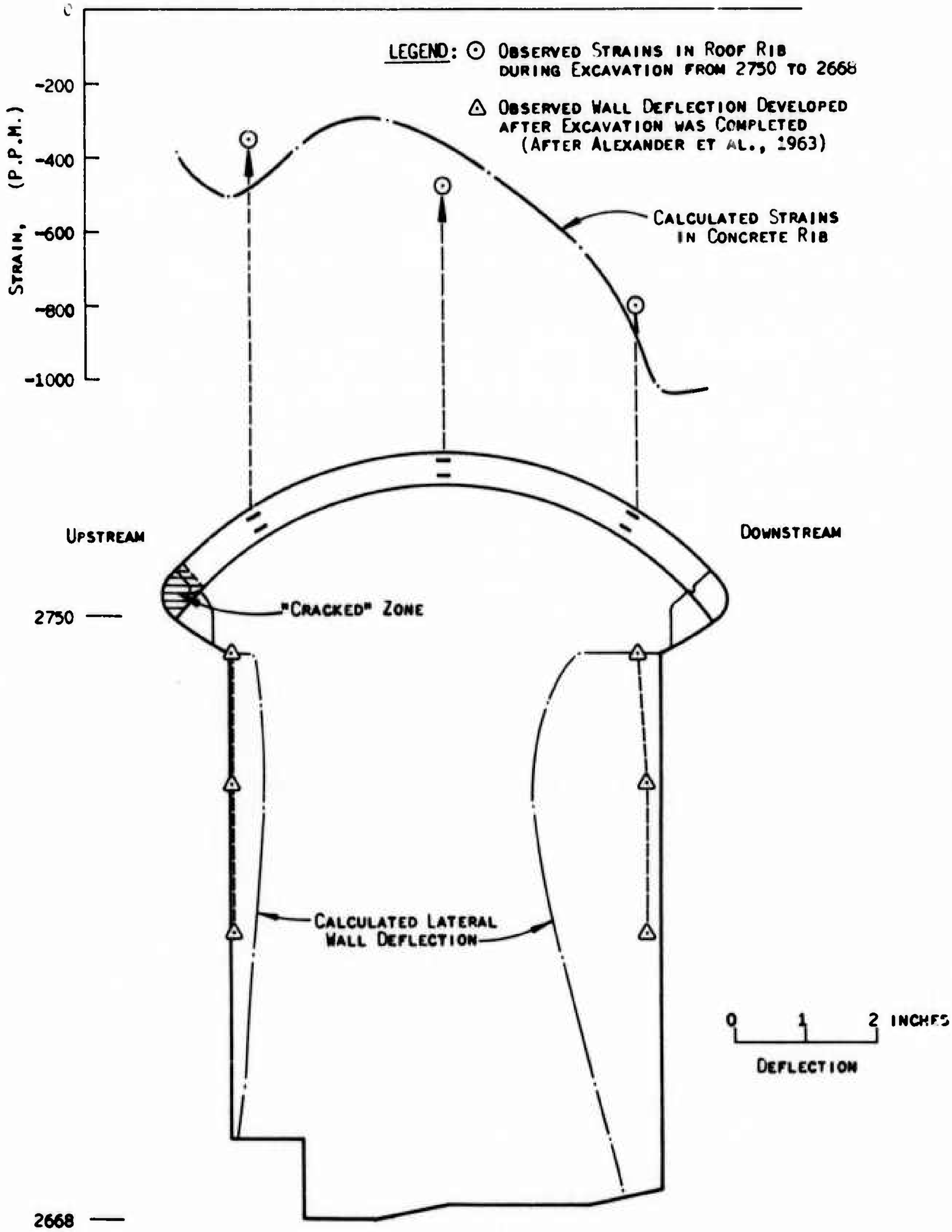


Fig. 57 Comparison of Observed and Predicted Behavior of Tumut I Underground Power Station

below the yield strength computed by the strength parameters prescribed, and the overall stability of the rock mass was maintained throughout the excavation. The results of the analysis indicated that a 50-foot section of the fault "A" at the upstream side immediately above the roof of the excavation failed in shear after the first stage of excavation (see Fig. 53). However, subsequent stress redistribution and installation of the concrete ribs prevented any further failure along the fault "A".

In summary, the analysis of the Tumut I underground machine hall indicated that the observed behavior of the excavation following a complicated excavation sequence could be predicted with a reasonable accuracy using the general computer program that has been developed. The major difficulty in analysis, however, lies in the determination of the distribution of major geologic discontinuities and their stress deformation characteristics.

ANALYSIS OF A ROCK TUNNEL, WASHINGTON D.C. METRO

Construction of rock tunnels for the Washington D.C. Subway (METRO) has been described by Mahar, Gau and Cording (1972), and Bawa and Bumanis (1972). Due to the detailed documentation of the geologic conditions and the performance data of the tunnel excavations, it is believed that the tunnel excavations of METRO would provide excellent case histories for this study to verify the reliability of the general computer program developed for evaluation of stability of underground excavations. General geologic, excavation and support conditions, and the results of instrumentation on rock movements during excavations of a rock tunnel were provided to us by Dr. E. J. Cording (1973) and J. W. Mahar (1973) of University of Illinois, Urbana. The general geologic, excavation and support conditions are briefly described in the following section.

General Geologic Conditions

The rock tunnel analyzed was driven through a foliated rock-schistose gneiss of quartz-mica composition. Average rock quality, defined as the RQD of the rock core, ranges between fair to good, except in the shear zones where rock quality is poor to very poor. The significant features of the rock are: (1) the continuous, smooth joint planes which form large rock blocks, and (2) highly continuous shear zones which parallel the rock foliation.

The major features of the rock structure observed at the rock tunnel consist of foliation, eight major shear zones, joints in seven principal orientations designated as Set Nos. 1, 2L, 2H, 3L, 3H, 4L, and 4H. The orientation of the joints is shown in Fig. 58. Rock foliation is moderate to well developed and strikes sub-parallel to the long axis of the tunnel (N15°W) and dips 60° to 70° west.

The major shear zones, all of which are oriented sub-parallel to rock foliation or Joint Set 1, strike 10° to 20° right of tunnel axis and dip 50° to 60° west. These zones are generally 1 to 5 feet in width and spaced 10 to 50 feet apart. The most prominent features within the shear zones are layers of gouge and/or broken rock. The layers are generally 1 to 6 inches wide, highly continuous and are planar to slightly wavy. The gouge consists of a sandy, clayey material and is generally 1 to 2 inches thick. Rock fragments are generally less than 2 inches in size. Slickensides occur throughout the gouge and along the boundaries of the rock fragments. The distribution of the shear zones as shown on the cross-section analyzed is given in Fig. 59.

Joints and slickensided joints designated as Joint Set 1 having the same altitude as the major shear zones are prominent throughout the tunnel. These joints are highly continuous,

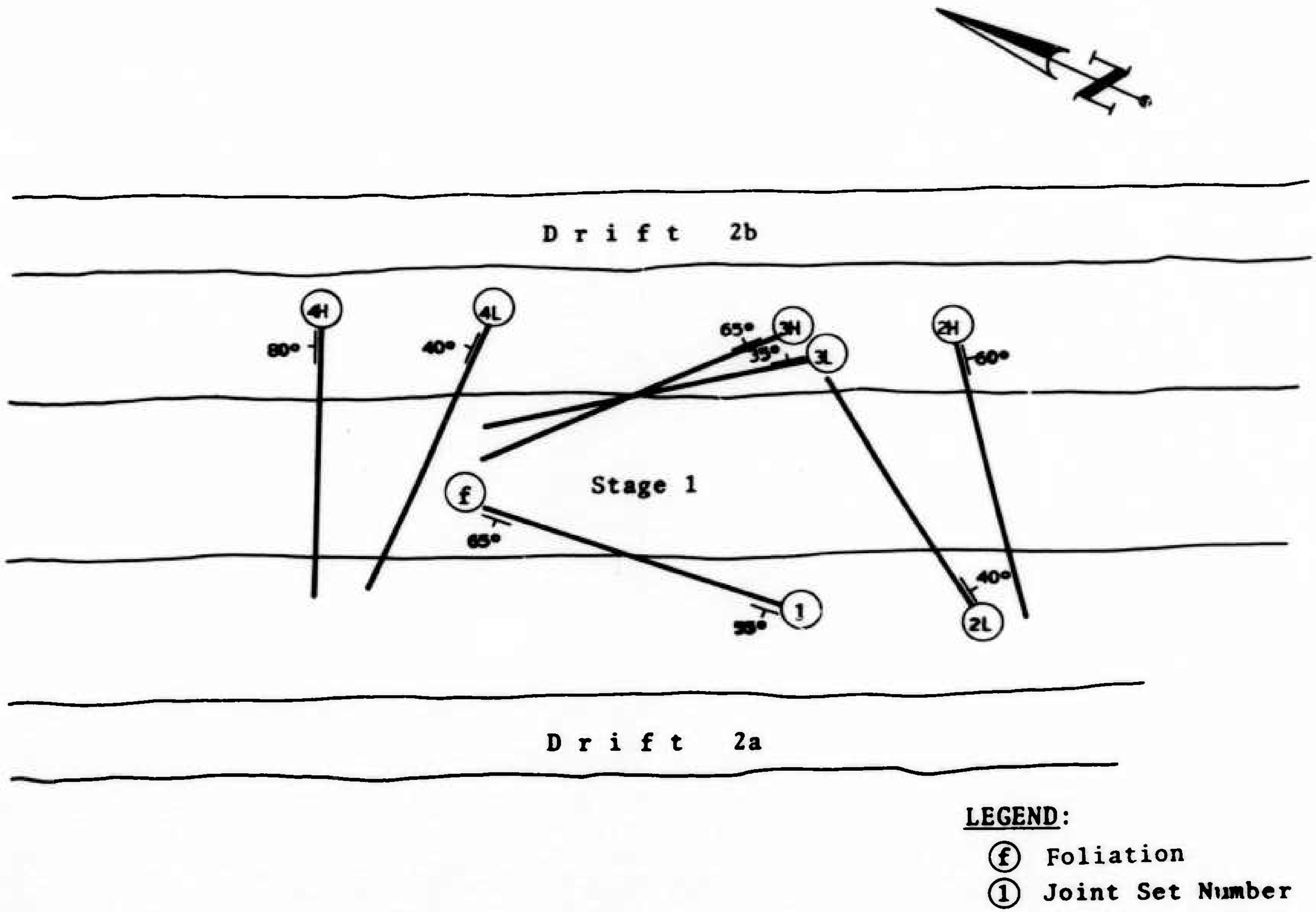


Fig. 58 Orientation of Major Geologic Discontinuities at a Rock Tunnel, Washington D.C. Metro (After Cording, 1973)

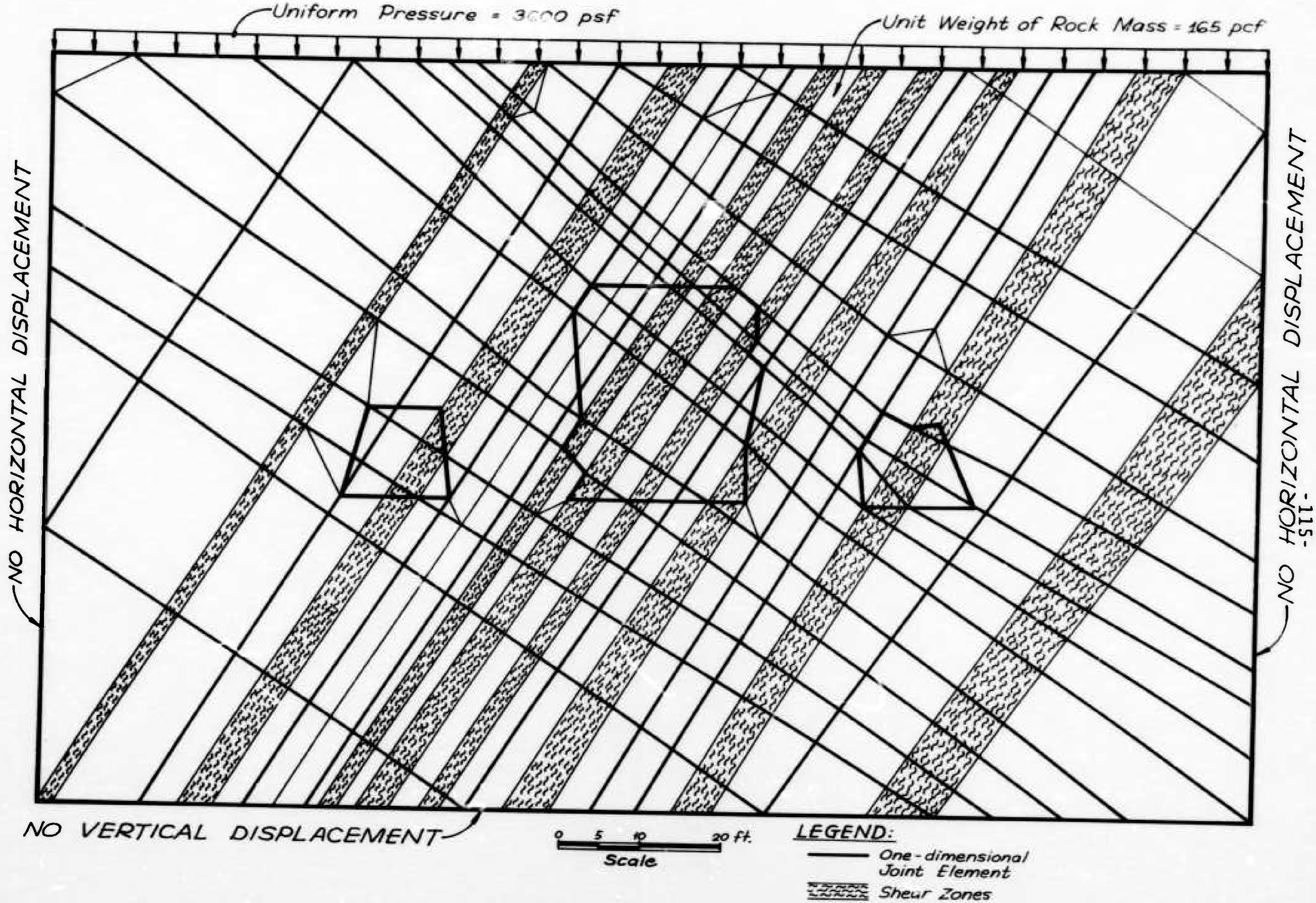


Fig. 59 Finite Element Idealization of a Rock Tunnel, Washington D.C. Metro

smooth and planar. The slickensided joints generally contain a talc and chlorite filling 1/8 to 1/2 inch thick. The average spacing of Joint Set 1 is 1 to 3 feet.

Joints in six other principal orientations are generally tight, planar, and continuous. These joints are not as commonly sheared or filled with gouge as the joints of Set No. 1. However, these joints may contain up to 1/4 inch of talc and chlorite and may be wet, particularly when the joints are located in the vicinity of foliation shear zones.

General Excavation and Support Conditions

The crown of the rock tunnel analyzed in this study is approximately 60 feet below the ground surface. The ground cover consists of a 30-foot-thick layer of soil immediately below the ground surface and a 30-foot-thick rock separated by 1 to 2 feet of weathered rock. The tunnel was driven in several stages: the pilot tunnel and Stages 1, 2a, and 2b.

The pilot tunnel was driven as a 6 foot by 8 foot exploratory drift through the full length of the station. After completion of the pilot tunnel, three rock bolts (24 feet long and 1-1/8 inches in diameter) were installed every 5 feet in the crown of the tunnel. Prior to driving Stage 1, bearing plates of most of the bolts were tightened and the first 6 feet of these bolts were grouted with cement.

The Stage 1 drift was driven as a box-shaped opening. The drift at the cross-section analyzed was a 18-foot-wide-by-24-foot-high opening. The drift was supported with shotcrete, rock bolts, and steel ribs. The heading of the Stage 1 excavation was initially supported with a layer of shotcrete having an average thickness of 2 inches. Steel ribs consisting of 14 WF posts

which did not provide lateral support to side walls and 8 WF beams were placed every 5 feet. After the steel ribs had been installed, four rock bolts were installed in the crown of the Stage 1 drift, two on either side of the pilot tunnel. These bolts spaced 5 feet of center and installed within 4 feet of the face were fully grouted with resin.

The 2a and 2b drifts were driven in approximately 10 feet by 10 feet openings in 5 feet advances. The thickness of the rock pillar between the Stage 1 drift and the side drifts at the cross-section analyzed was approximately 13 feet.

Idealization of Tunnel Excavation

Finite Element Idealization - The finite element idealization of the tunnel excavation is shown in Fig. 59. The idealization of the geologic profile includes a number of shear zones and Joint Sets 1, 3L, and 3H as projected from the geologic map on to the cross-section analyzed. Joint Sets 2L, 2H, 4L, and 4H were not present on the cross-section because the strikes of these joints are approximately perpendicular to the axis of the tunnel. The excavation was idealized as a 3-stage excavation: Stages 1, 2a, 2b. The pilot drift was considered to be a part of Stage 1 excavation. During excavation of the pilot drift and Stage 1 drift, installation of seven rock bolts at the crown was simulated by application of equivalent nodal point forces of 4000 lbs/ft at the bearing plate and the anchor of each rock bolt. Idealization of the grouted rock bolts may be improved by considering the rock bolts as one-dimensional bar elements with the same material properties as those of the rock bolts. The 2-inch-thick layer of shotcrete and the steel ribs were not simulated in the analysis. As noted in Fig. 59, the upper 30-feet-thick layer of soils immediately below the ground surface was replaced by applying a uniform pressure equivalent to the overburden pressure of 3600 psf on the upper boundary of the rock cover in order to minimize the number of

elements and nodes required in the idealization. The total number of elements and nodes used were 863 and 986, respectively.

Initial State of Stress - No stress measurements were made at the site. However, Cording (1973) expressed the opinion that there is no high horizontal stress existing at the site and the vertical stress is approximately equal to the overburden pressure. In the parametric study to be described later, two initial states of stress were assumed. The vertical stress was assumed equal to the overburden pressure and the ratios of horizontal to vertical stress were 0.6 and 0.8 for two initial stress conditions.

Material Properties - No test data were available for determination of material properties for intact rocks, shear zones and joints present in the geologic profile. However, reasonable values were assumed for deformation moduli and strength parameters for the intact rock and shear zones, and for deformability and shear strength of the joints, and a parametric study conducted to select the properties for more detailed analysis. The assumed material properties used in the parametric study (Cases A, B and C) are summarized in Table 5. The deformation modulus of the intact rock was assumed to be 1,000,000 psi for all three cases, and 1,000,000 psf for the shear zones. The values of the Poisson's ratio were assumed to be 0.15 and 0.3 for the intact rock and the shear zones, respectively. For Cases A and B, the normal stiffness of the joints were assumed to be 1×10^7 pci and the shear stiffness to be 5×10^6 pci, and the angle of shearing resistance equal to 25 degrees with no cohesion. The normal and shear stiffnesses for the joints for Case C were 100% higher than those for Cases A and B and the angle of shearing resistance was reduced to 20 degrees.

Table 5. Material Properties and Stress Conditions
Assumed in Parametric Study of a Rock Tunnel
Washington D.C. Metro

Case	Intact Rock				Shear Zones				Joints				Initial Stresses
	E psf	ν	C psf	ϕ deg.	E psf	ν	C psf	ϕ deg.	K_n pcf	K_s pcf	C_j psf	ϕ_j deg.	
A	$1.44x10^8$	0.15	$7.2x10^5$	45	$1.x10^6$	0.3	0.	35	$1.x10^7$	$5.x10^6$	0.	25	$\sigma_v = \sigma_o$ K = 0.8
B	$1.44x10^8$	0.15	$7.2x10^5$	45	$1.x10^6$	0.3	0.	35	$1.x10^7$	$5.x10^6$	0.	25	$\sigma_v = \sigma_o$ K = 0.6
C	$1.44x10^8$	0.15	$7.2x10^5$	45	$1.x10^6$	0.3	0.	35	$2x10^7$	$1.x10^7$	0.	20	$\sigma_v = \sigma_o$ K = 0.8

E = Modulus
ν = Poisson's Ratio
K_n = Normal Stiffness for Joint Elements
K_s = Shear for Joint Elements
ϕ = Angle of Friction
C = Cohesion

σ_v = Vertical Stress
$\sigma_h = K \sigma_v$ = Horizontal Stress
σ_o = Overburden Pressure

Presentation and Discussion of Results

Three cases were analyzed in the parametric study to examine the effect of the initial stress conditions and the deformability and strength properties of the joints on the computed movements surrounding the tunnel due to Stage 1 excavation. The results of the parametric study are summarized in Table 6. Comparing Cases A and B for the elastic solution, when the ratio of the horizontal to vertical stress decreases from 0.8 (Case A) to 0.6 (Case B), the movement at the crown increases 15% while the movement on the side walls decrease 20% to 25%. In the non-linear solution in which effects of joint and shear zone failure were considered, the movements at the crown and the east wall for Case B were 10% higher than those for Case A, indicating that more elements would fail under low horizontal stress conditions. Comparing Cases A and C, it may be noted that while the joint stiffnesses increased 100%, the movements decreased 10 to 20% indicating the presence of the shear zones might have significant effects on the behavior of the excavation. It is interesting to note that for all three cases, the movement on the west wall remained about the same for both elastic and non-linear solutions.

The detailed analysis was conducted using the material properties and the initial stress conditions assumed for Case A. The results of the detailed analysis of the 3-stage excavation are summarized in Table 7 and Figs. 60 and 61. Table 7 and Fig. 60 summarize the computed movements at the crown and side walls of the Stage 1 drift due to each stage of excavation. The computed movement at the crown increased from 0.27 inches due to the Stage 1 excavation to 0.41 inches at the end of the Stage 2b excavation. The total movement observed at the crown was 0.42 inches. The computed movement on the east wall increased from 0.52 inches due to the Stage 1 excavation to 0.61 inches at the end of Stage 2b excavation. The total movement

Table 6. Summary of Results of Parametric Study on a Rock Tunnel, Washington, D. C. Metro

Case	Computed Movements Due to Stage 1 Excavation on the Excavated Surface in the Extensometer Directions					
	Elastic Solution			Nonlinear Solution		
	DX 3,4 Crown	DX 20 East Wall	DX 21 West Wall	DX 3,4 Crown	DX 20 East Wall	DX 21 West Wall
A	0.20 in.	0.48 in.	0.24 in.	0.27 in.	0.52 in.	0.24 in.
B	0.23 in.	0.37 in.	0.17 in.	0.31 in.	0.57 in.	0.18 in.
C	0.15 in.	0.43 in.	0.19 in.	0.21 in.	0.49 in.	0.20 in.

Table 7. Summary of Results of Analysis of a Rock Tunnel, Washington D. C. Metro. -Computed and Observed Movements on the Excavated Surface in the Extensometer Directions (Case A)

Location	Stage 1 Excavation		Side Drift 2a	Side Drift 2b	Observed (final)	One-Step Elastic Solution
	Elastic Solution	Nonlinear Solution				
DX 3,4 Crown	0.20 in.	0.27 in.	0.34 in.	0.41 in.	0.42 in.	0.26 in.
DX 20 East Wall	0.48 in.	0.52 in.	0.55 in.	0.61 in.	0.67 in.	0.51 in.
DX 21 West Wall	0.24 in.	0.24 in.	0.21 in.	0.22 in.	*0.85 in.	0.19 in.

*Due to loosening of a rock block, large movement was only observed near the excavated surface.

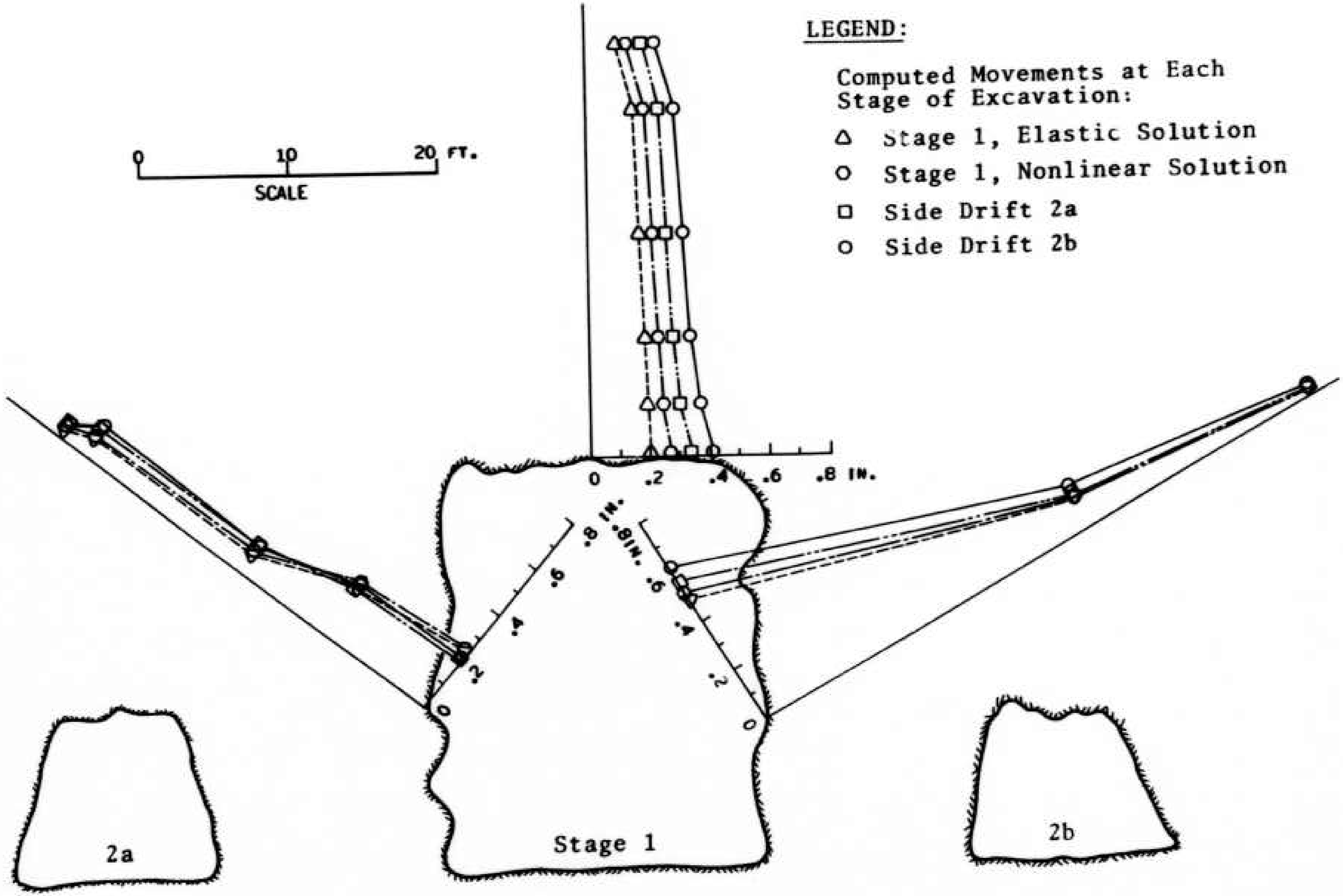


Fig. 60 Computed Movements During Simulation of 3-Stage Excavation of a Rock Tunnel, Washington D.C. Metro

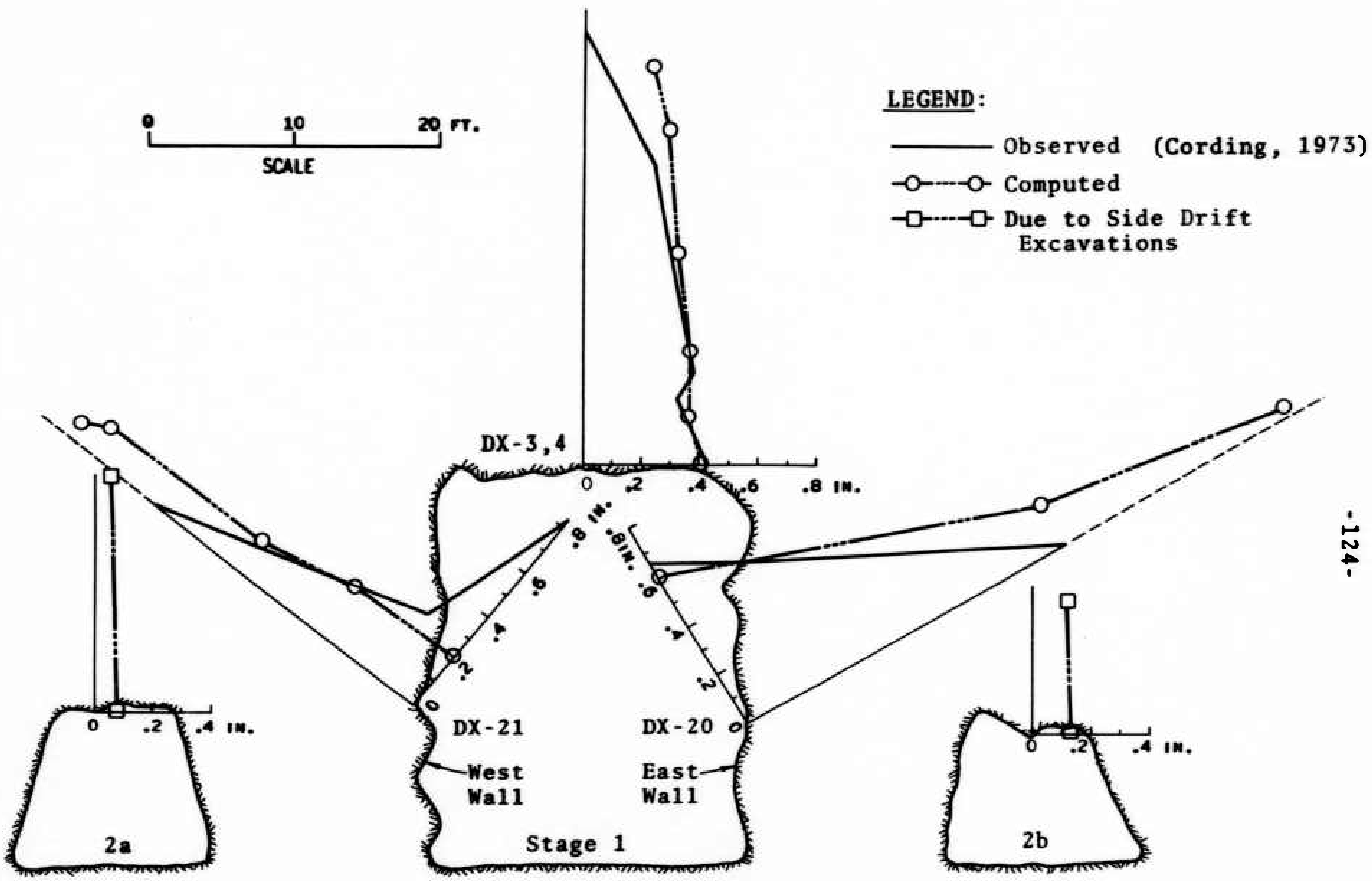


Fig. 61 Computed and Observed Movements During Excavation of a Rock Tunnel, Washington D.C. Metro

observed on the east wall was 0.67 inches. The computed movement on the west wall was 0.24 inches due to the Stage 1 excavation and decreased slightly at the end of the Stage 2b excavation. Also shown on Table 7 are the movements computed from the elastic solution of the 3-stage excavation. The crown movement computed was 0.26 inches as compared to 0.41 inches computed from the nonlinear solution. The east wall movement computed was 0.51 inches as compared to 0.61 inches computed from nonlinear solution. The west wall movement computed was slightly less than that computed from the nonlinear solution.

Figure 61 illustrates the comparison between the computed and observed movements due to the excavation of the rock tunnel analyzed. The computed total movements at the excavated surface are generally in close agreement with those observed, except the one on the west wall for which the larger movement was measured near the surface due to loosening of near-surface rock blocks. However, a significant portion of movements was computed at the anchor points of the extensometers, indicating that the computed movements within the instrumented rock zones are less than those measured. It is expected that the results of the analysis could be improved and would tend to indicate failure and loosening of larger portions of shear zones and joint blocks, similar to the behavior observed in the field, if lower values of the friction angles for shear zones and joints, and higher values of modulus for shear zones are used in the analysis.

The results of the analysis of the rock tunnel of the Washington D.C. METRO indicate that the behavior of the rock mass formed by various continuous joint sets and shear zones due to underground excavations could be predicted with a reasonable degree of accuracy. Although the properties of significant geologic features; e.g., joints and shear zones, present in the rock mass were not available for the analysis, it was possible to determine reasonable material properties from a parametric

study and the observed performance of the underground excavation. The analysis was considered to be useful in understanding the behavior of the excavation in that particular rock mass. The material properties utilized can be employed to predict the response of the rock mass around further tunnel excavations in similar geologic conditions.

CONCLUSIONS AND RECOMMENDATIONS

For the purpose of evaluating the capabilities and illustrating the use of the computer code developed, studies on a number of well documented model tests and case histories of underground openings were analyzed. These included the analysis of model tests on lined and unlined openings in jointed rock, and the analysis of Tumut I underground power station and a rock tunnel of Washington, D.C. (METRO) Subway.

The conclusions drawn from this evaluation are as follows:

1. The major problem in utilizing the computer code developed to the analysis of practical problems is the lack of information on the properties of the rock mass especially with respect to the location and deformation characteristics of geological discontinuities.

2. The properties of geological discontinuities can be determined through a parametric study using various combinations of material properties and comparing the results with limited aspects of observed performance.

3. Utilizing the properties selected in (2) additional aspects of the behavior of excavation in rock can be predicted with reasonable accuracy.

4. The program can be calibrated in terms of material properties in the initial phases of an excavation and then utilized to predict future performance.

5. The results of this study indicate that if the location and properties of geologic discontinuities can be defined then the computer code developed can be a valuable aid to the design of excavations in rock.

Recommendations for Future Research

Although studies of a limited number of the case histories indicated that the analytical models developed under this contract could predict the behavior of underground openings with a reasonable accuracy, more case history studies are required to fully evaluate the capabilities and the improvements required for the computer code developed.

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APPENDIX A

A COMBINED COMPUTER PROGRAM USING FINITE ELEMENT TECHNIQUES FOR ELASTO-PLASTIC, JOINT PERTURBATION AND NO TENSION ANALYSIS OF SEQUENTIAL EXCAVATION AND CONSTRUCTION OF UNDERGROUND OPENINGS IN ROCK

A COMBINED COMPUTER PROGRAM USING FINITE ELEMENT TECHNIQUES FOR ELASTO-PLASTIC JOINT PERTURBATION AND NO TENSION ANALYSIS OF SEQUENTIAL EXCAVATION AND CONSTRUCTION OF UNDERGROUND OPENINGS IN ROCK

Identification

The program which consists of a main program and 15 subroutines (NPSTRS, STIFF, MODIFY, QUAD, TRISTF, JTSTIF, BANSOL, STRESS, INITST, PRINST, LOAD, JTSTR, EPLAST, STRSTR, NPFORC) is a modification of the computer program developed and documented in the report, "A Theoretical Method for Evaluating Stability of Openings in Rock," by C. Y. Chang and K. Nair, U.S. Bureau of Mines Contract No. HO210046 (April 1972). The major improvements and modifications have already been discussed elsewhere in this report.

Purpose

The combined program has been developed to take into account the actual construction and excavation sequences which are important factors to be considered, especially in non-linear materials. The rock mass may consist of joints, faults, bedding planes and other geologic discontinuities. The intact rock may be incapable of sustaining any tensile load and exhibit elastic-perfectly plastic behavior.

Sequence of Operation

(a) The main program handles the initial input and monitors the calling of the subroutines in a specified order as shown in Fig. A1. If specified for the last iteration of the last step in analysis stresses, excess stresses to be redistributed, nodal point displacements and yield functions are punched onto cards to be used for restarting computation. This allows one to monitor the results as analysis proceeds, without loss of computer time.

(b) Subroutine NPSTRS computes stresses at nodes on the excavated boundary from stresses in surrounding elements.

(c) Subroutine STIFF assembles the general stiffness matrix for the entire structure, adds the concentrated loads at the nodal points, and modifies the stiffness matrix for the boundary conditions.

(d) In Subroutine QUAD the stiffness of a two-dimensional element is formulated.

(e) Subroutine JTSTIF forms the stiffness matrix for each joint element.

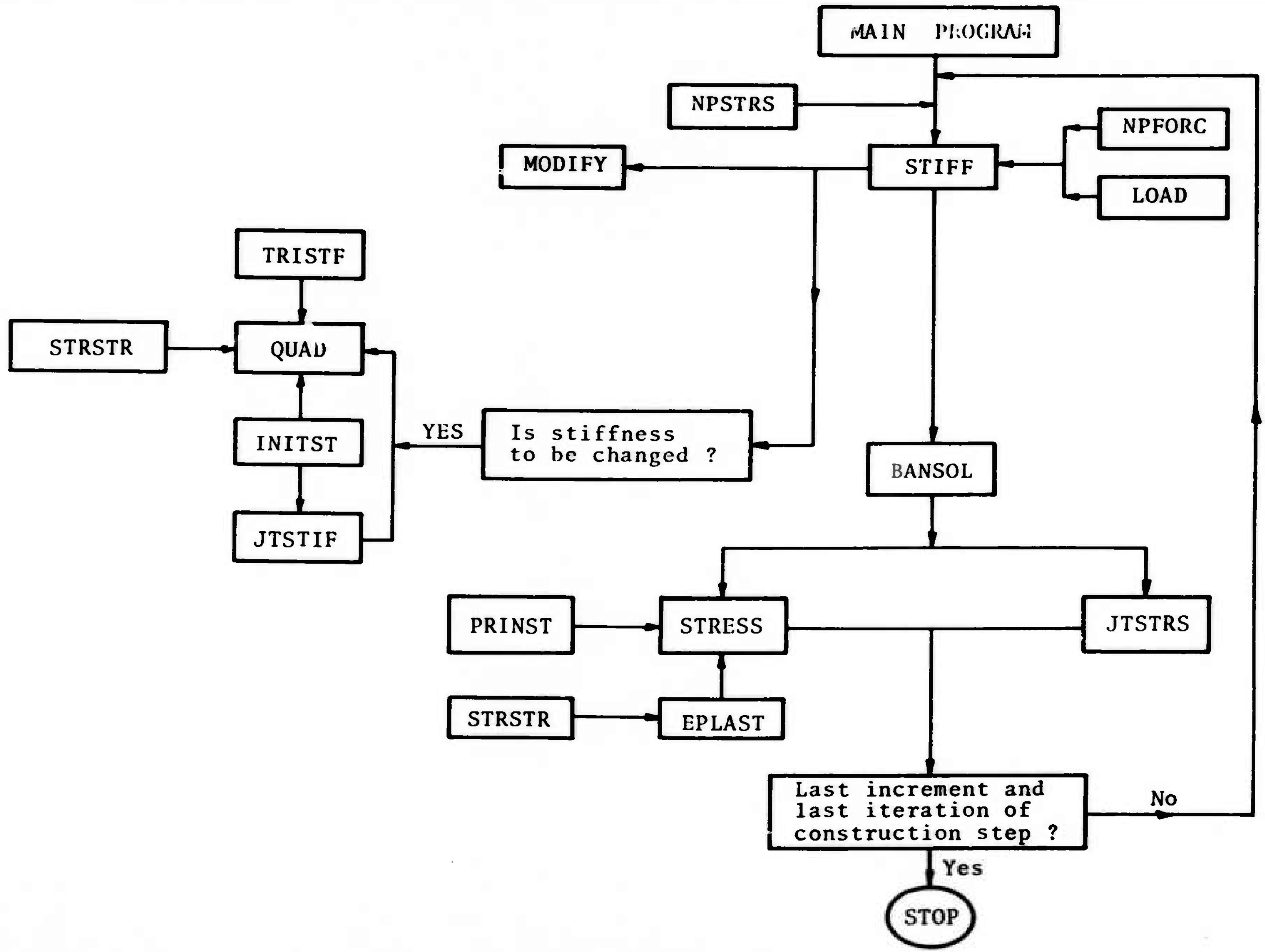

FIGURE A1 - SIMPLIFIED FLOW DIAGRAM SHOWING SEQUENCE OF OPERATION OF ALL SURROUTINES

(f) Subroutine TRISTF forms the stiffness matrix for triangular sub-elements and, if specified, element loads due to gravity are calculated.

(g) Subroutine MODIFY modifies the stiffness matrix for the boundary conditions.

(h) Subroutine LOAD calculates equilibrating nodal point forces due to gravity, if specified, and for excess stresses computed in Subroutine EPLAST for elasto-plastic and/or no tension materials.

(i) Subroutine BANSOL solves the simultaneous equations representing the structural stiffness matrix and the structural load vector for nodal point displacements.

(j) Subroutine STRESS calculates incremental stresses and strains, cumulates stresses, and prints stresses and strains for two-dimensional elements.

(k) Subroutine JTSTRS calculates and prints normal and tangential displacements (cumulative and incremental) and excess normal and tangential stresses to be redistributed by comparing stress with strength for joint elements. The equilibrating nodal point forces are also computed from the excess stresses and stored for the next iteration.

(l) Subroutine EPLAST calculates yield functions and elasto-plastic stress-strain relation for those two-dimensional elements in yield. The excess stresses to be redistributed are computed as a difference between changes in stress calculated from the elastic stress-strain relation and those calculated from the elasto-plastic stress-strain relation.

(m) Subroutine INITST generates initial stresses under free-field conditions.

(n) Subroutine PRINST calculates magnitudes and directions of the principal stresses and strains.

(o) In Subroutine STRSTR the constitutive law for the material is formulated.

(p) In Subroutine NPFORC the nodal point forces due to boundary pressures are calculated.

Output

The data describing the finite element configuration, the material properties and pressures applied to the excavated face to simulate excavation for the opening are printed after being read. Nodal point displacements (incremental and cumulative), stresses, strains

and yield functions for two-dimensional elements; normal and tangential stresses, normal and tangential displacements (incremental and cumulative) for joint elements, are printed after each increment or iteration. If specified, for the last iteration of the last increment in the last construction step specified in an analysis, stresses, strains and excess stresses to be redistributed for two-dimensional elements, normal and tangential stresses for joint elements, nodal point displacements and yield functions for two-dimensional elements are punched onto cards to be used for restarting computation.

Input Data Procedure

1st CARD TYPE: FORMAT (8A10) (One Card)

Cols 2-80 Identifying information to be printed with results.

2nd CARD TYPE: FORMAT (3I5, 2F10.2, I5, 2F10.5, 4I5) (One Card)

Cols. 1-5	NUMNP	- Number of nodal points (maximum 999)
6-10	NUMEL	- Number of elements (maximum 900)
11-15	NUMMAT	- Number of different materials (maximum 12)
16-25	ACELX	- Acceleration in X-direction
26-35	ACELY	- Acceleration in Y-direction
36-40	NRES*	- = -1, Residual stresses generated from which residual load is calculated. = 0, Residual stresses generated, but residual load is zero. = 1, Residual stresses read as input, from which residual load is generated. = 2, Residual stresses read as input, but residual load is zero. = 3, Residual stresses will be computed for the purpose of calculating nodal forces along excavated boundary.
41-50	REFPR	- Vertical stress at the reference point.

*If NREAD = 1, NRES should not be greater than zero.
If NRES = -1 or 1, gravity turn-on analysis is performed.

51-60 DEPTH - Y-ordinate at the reference point.

61-65 NANALY - = 0, Analysis using stress transfer techniques with constant initial stiffness.

NANALY - = 1, Analysis using stress transfer techniques with updating element stiffness at each increment of load.

66-70 NCONST - Total number of construction steps simulated in analysis.

71-75 NPUNCH - = 0, Data will not be punched out at the last iteration.

= 1, Data will be punched out at the last iteration of the last increment at the last construction step.

76-80 NREAD - = 0, No data from previous computation will be read as input.

= 1, Data from last increment are read as input.

<u>3rd CARD TYPE</u>: FORMAT (2I5) (One Card)

Cols. 1-5 MJØINT - Total number of material types for joints (maximum 12)

6-10 MTENS - Total number of material types that can sustain tension.

<u>4th CARD TYPE</u>: FORMAT (16I5) (Omit this card if MJOINT = 0 on 3rd Card Type)

Cols. 1-5 MJNT(I) - Material type number for joint elements.

6-10 Same

11-15 ----

<u>5th CARD TYPE</u>: FORMAT (16I5) (Omit this card if MTENS = 0 on 3rd Card Type)

Cols. 1-5 MNTEN(I) - Material type number which can sustain tension.

6-10 Same

11-15 ----

6th CARD TYPE: FORMAT (I5, 2F10.5) (One Card)

Cols. 1-5	MTYPE	- Material type number.
6-15	RO(MTYPE)	- Mass density of this material type.
16-20	AKO(MTYPE)	- Ratio of horizontal to vertical stress under initial stress conditions.

7th CARD TYPE: FORMAT (8F10.5, I5)

Cols. 1-10	E(1, MTYPE)	- Tensile strength for normal materials or normal stiffness for joint materials.
11-20	E(2, MTYPE)	- Modulus in compression for normal materials or shear stiffness for joint materials.
21-30	E(3, MTYPE)	- Poisson's ratio for normal materials or cohesion for joint materials.
31-40	E(4, MTYPE)	- Modulus in tension for normal materials or angle of friction for joint materials (degrees)
41-50	E(5, MTYPE)	- Cohesion for normal materials or maximum allowable closure (input as negative) for joint materials.
51-60	E(6, MTYPE)	- Angle of friction for normal materials (degrees)
61-65	NTEST	- Type of Test = 0, if c and ϕ obtained from triaxial test = 1, if c and ϕ obtained from plane strain test.
66-75	CRAC(MTYPE)	- Fraction of tensile strength which the material is allowed to take after tension failure.

Repeat 6th and 7th card types for all material types.

8th CARD TYPE: FORMAT (I5, F5.0, 4F10.0) (One card for each nodal point)

Cols. 1-5	N	- Nodal point number.

	6-10	CODE (N)	- Number which indicates if displacements or forces are to be specified.
			= 0 UR is the specified X-load and UZ is the specified Y-load
			= 1 UR is the specified X-displacement and UZ is the specified Y-load
			= 2 UR is the specified X-load and UZ is the specified Y-displacement
			= 3 UR is the specified X-displacement and UZ is the specified Y-displacement
Cols.	11-20	R(N)	- X-ordinate
	21-30	Z(N)	- Y-ordinate
	31-40	UR(N)	- X-load or displacement
	41-50	UZ(N)	- Y-load or displacement

Nodal points must be numbered in sequence. If nodal point numbers are omitted, those omitted are generated automatically at equal spacings, between those specified and CODE(N) is assigned zero. The first and last nodal points must be specified.

<u>9th CARD TYPE</u>: FORMAT (6I5) (One card for each element)

Cols.	1-5	M	- Element number
	6-10	IX(M,1)	- Nodal point I
	11-15	IX(M,2)	- Nodal point J
	16-20	IX(M,3)	- Nodal point K
	21-25	IX(M,4)	- Nodal point L
	26-30	IX(M,5)	- Material number

The nodal point numbers must be numbered consecutively proceeding counterclockwise around the elements. The nodal point numbers for any element must not differ by more than 44. If element numbers are omitted, those missing will be generated by incrementing the element number and each nodal point number (I, J, K and L) by one, and assigning the same material number as the last element specified. The first and last elements must be specified.

Triangular elements are also permissible, and are identified by repeating the last nodal point number (i.e., I, J, K, K). For joint elements, nodal points must be numbered I, J, K, L counterclockwise proceeding along length of joint from I to J and along length from K to L. Nodal points I and L (J and K) have different numbers but identical coordinates.

One-dimensional elements are also permissible and are identified by the node number sequence (I, J, J, I).

10th CARD TYPE: FORMAT (I5, 4E15.5) (One card for each element)

Cols. 1-5	N	- Element number
6-20	STRS(N,1)	- Initial stress σ_x
21-35	STRS(N,2)	- Initial stress σ_y
36-50	STRS(N,3)	- Initial stress τ_{xy}
51-65	STRS(N,4)	- Initial stress σ_z

This card type is neglected if NRES $\leq$ 0 or NRES=3 on the 2nd card type.

11th CARD TYPE: FORMAT (I5)

Cols. 1-5	NJT	- Total number of joint elements.

12th CARD TYPE: FORMAT (2F20.5, I5)

Cols. 1-20	FN(I)	- Normal stress across joint element NEJT(I)
21-40	FT(I)	- Tangential stress across joint element NEJT(I)
41-45	NEJT(I)	- Element number for joint element

Repeat 12th card type for all joint elements.

11th and 12th card types are neglected in MJOINT = 0 on the 3rd card type or NRES $\leq$ 0 or NRES = 3 on the 2nd card type.

13th CARD TYPE: (3I5)

Cols. 1-5	NCPSNT	- Present construction sequence number
6-10	NPRSNT	- Present load increment number
11-15	ITPSNT	- Present iteration number

14th CARD TYPE: (8A10)

Cols. 2-80	TITLE	- The information regarding this construction sequence number to be printed out.

15th CARD TYPE: (8I5)

Cols. 1-5 NERSP - Number of elements to be excavated and/or for structural support.

6-10 NUMPC - Number of pressure cards.

11-15 NPF - Number of nodal points at which forces are applied.

16-20 NPCAV - Number of nodal points along current excavated face.

21-25 NP - Number of load increments by which boundary pressures and pressures computed for excavation simulation will be divided into small load increments. The magnitude of increment is to be specified on the card type.

26-30 NGLD - = 0 Not a gravity turn-on analysis step.

= 1 Present step is gravity turn-on to obtain initial stresses.

31-35 MTRM - Material type for excavated elements.

36-40* NSPT - Index indicating if there is any element to be converted to liner for current construction step.

= 0 if there is no element to be converted to liner

= 1 if there are some elements to be converted to liner

16th CARD TYPE: (16I5) (Neglected if NERSP = 0 on the 15th card type)

This card type carries the numbers of all elements removed or added and their changed material type. As many cards as are required to be provided.

Cols. 1-5 NE(I) - Element number.

6-10 MT(I) - New material type for this element.

11-15 MSP(I) - Material type of liner if the element is to be "excavated" first and then converted to the liner. MSP(I)=MT(I) if NSPT=0 on the 15th card type or the element not to be "converted" to the liner.

16-20
21-25 } - Same as above.
26-30

17th CARD TYPE: FORMAT (16I5)

Cols. 1-5 IJBC(L,1) - Nodal point number I along the boundary IJ where the boundary pressure is applied.

*See Table A-1 for significance of the index.

6-10 IJBC (L,2) - Nodal point J along the boundary IJ.

11-15, 16-20 - Same as above; two nodal point numbers for each boundary where the boundary pressure is applied.

21-25, 26-30 - ----

As shown in Fig. A2, nodal points I and J must be ordered in counterclockwise order about centroid of element on which the pressure is applied.

18th CARD TYPE: FORMAT (I5)

Cols. 1-5 NPBCP - Number of nodal points along the boundary where the stresses are applied.

19th CARD TYPE: FORMAT (I5, 3F10.0)

Cols. 1-5 NPBC(M) - Nodal point number where the boundary pressure is applied.

6-15 PSCA(M,1) - σ_x at nodal point.

16-25 PSCA(M,2) - σ_y at nodal point.

26-35 PSCA(M,3) - τ_{xy} at nodal point.

Stresses (σ_x, σ_y and τ_{xy}) are shown positive in Fig. A2. To simulate excavation, these stresses are equal in magnitude and opposite in direction to the initial stresses at the nodal point. As shown in Fig. A2, stresses (σ_x, σ_y and τ_{xy}) at the nodal points are converted to normal and shear stresses on the boundary. Then the nodal point forces are calculated from the normal and shear stresses along the boundary assuming linear stress distribution along the boundary. The normal and shear stresses are shown positive in Fig. A2. These cards are provided to take into account externally applied stresses.

17th, 18th and 19th card types are neglected if NUMPC = 0 on the 15th card type.

20th CARD TYPE: FORMAT (I5, F5.0, 2F10.0)

These cards read the information regarding the change in the boundary conditions for all the nodal points involved.

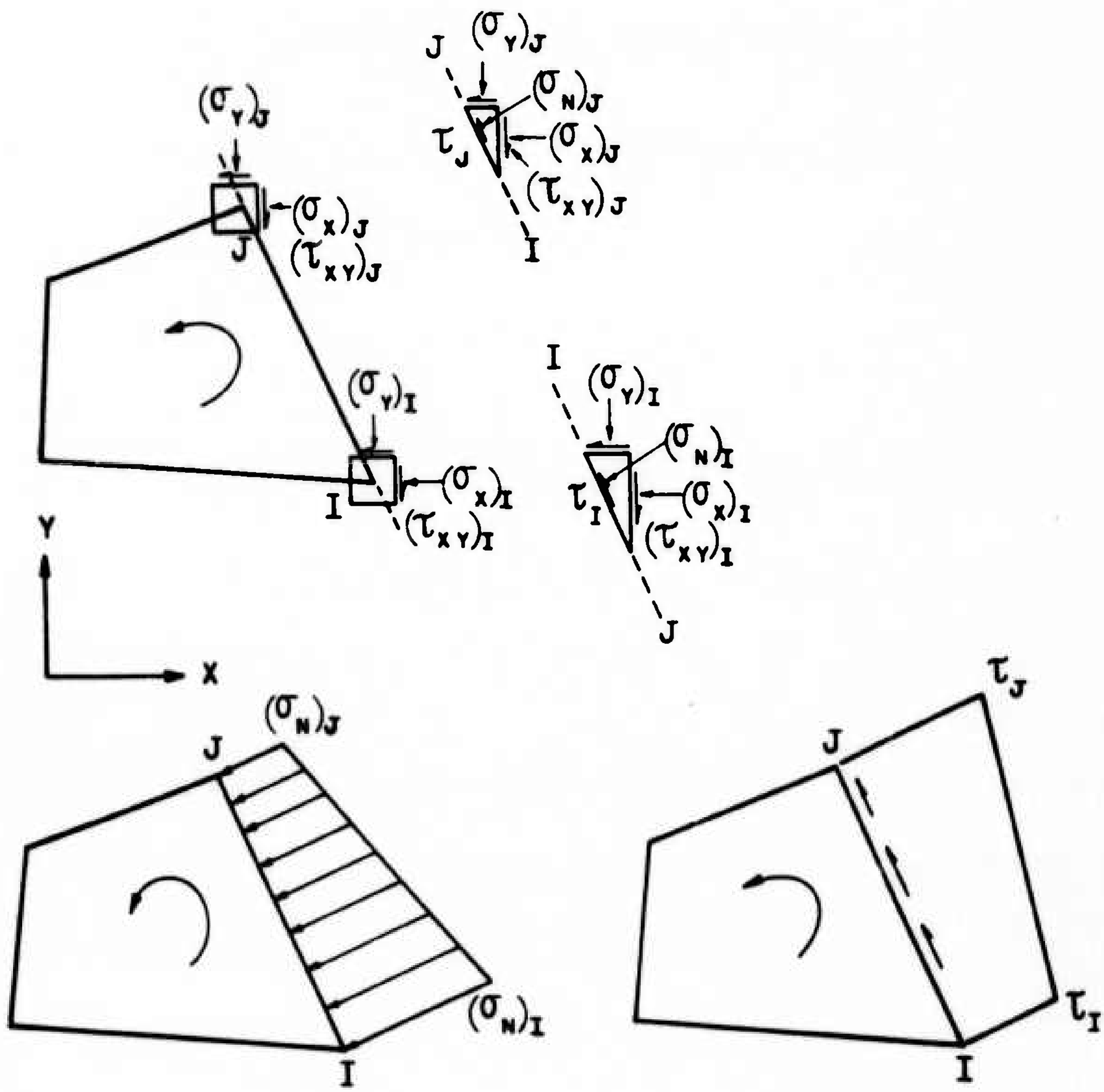

FIG. A2 - SIGN CONVENTION FOR BOUNDARY PRESSURE

Cols. 1-5 N - Number of nodal point.

6-10 CODE(N) - Type of nodal point.

11-20 UR(N) - Load or displacement in X-direction.

21-30 UZ(N) - Load or displacement in Y-direction.

These cards are omitted if NPF = 0 on the 15th card type.

21st CARD TYPE: FORMAT (5I5)

Cols. 1-5 NS(I) - Nodal point number at which nodal stresses are to be computed.

6-10 NSEL(I,1) - First interpolation element number.

11-15 NSEL(I,2) - Second interpolation element number.

16-20 NSEL(I,3) - Third interpolation element number.

21-25 NSEL(I,4) - Fourth interpolation element number.

Repeat for all nodal points at which nodal stresses are to be computed. These are provided to simulate excavation.

The mid-points of no three of the four interpolation elements may lie on a horizontal or vertical line. These elements should be read in a criss-crossed fashion as shown in Fig. A3. The centroids of the first and second elements should not lie on a vertical line.

These cards are omitted if NPCAV is zero on the 15th card type.

22nd CARD TYPE: FORMAT (I5)

Cols. 1-5 NCAVPC - Number of pressure cards on the cavity face to simulate excavation.

23rd CARD TYPE: FORMAT (16I5)

Cols. 1-5 IJBCA (L,1) - Nodal point I along boundary IJ where pressure is applied.

6-10 IJBCA (L,2) - Nodal point J along boundary IJ.

11-15, 16-20 - Same as above. Two nodal point numbers for each boundary where the boundary pressure applied.

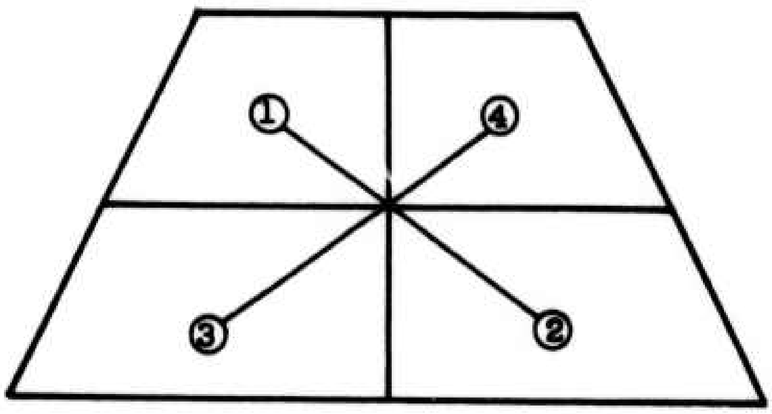

FIG. A3 - SEQUENCE FOR READING IN
INTERPOLATION ELEMENTS

21-25

\- ----

26-30

22nd and 23rd card types are neglected if NPCAV = 0 on the 15th card type.

24th CARD TYPE: FORMAT (I5, F10.5)

Cols. 1-5 ITN(N) - Number of iteration for Nth increment.

6-1: PRATIO(N) - Fraction of total pressure applied for Nth increment. The sum of PRATIO (N) for all increments should be equal to 1.

Repeat for each loading increment.

25th CARD TYPE: Binary data cards for all elements which are punched out at the end of the previous computer run.

STRS(N,1) - σ_x

STRS(N,2) - σ_y

STRS(N,3) - τ_{xy}

SEP(N,1) - Excess stress σ_x'

SEP(N,2) - Excess stress σ_y'

SEP(N,3) - Excess stress τ_{xy}'

STRS(N,4) - σ_z

MTAG(N) - An index indicating if the element has failed in tension or yielded under compressive stress field.
If MTAG(N) = 1, the element has not failed in tension or compressive stress field
MTAG(N) = 2, the element has failed in tension in one direction only.
MTAG(N) = 3, the element has failed in tension in two orthogonal directions.
MTAG(N) = 4, the element has not failed in tension but yielded under compressive stress field.
MTAG(N) = 5, the element has failed in tension in one principal stress direction, and subsequently yielded under compressive stress field.
MTAG(N) = 6, the element has failed in tension in two orthogonal directions, and subsequently yielded under compressive stress field.

<u>26th CARD TYPE</u>: Binary data cards for all joint elements, neglected if MJOINT = 0.

NJT - Total number of joint elements
FN(N) - Normal stress across joint element
FT(N) - Tangential stress across joint element

25th and 26th card types are neglected if NREAD = 0 on the 2nd card type, i.e., <u>these cards are only needed to restart the computation</u>.

<u>27th CARD TYPE</u>: Binary data cards for all nodal point displacements, yield function, and all strain components for all elements.

DISP(N,1) - X-displacement for node N

DISP(N,2) - Y-displacement for node N

FY(N) - Yield function for element N

STRN(N,1) - Strain component, ε_x, for element N.

STRN(N,2) - Strain component, ε_y, for element N.

STRN(N,3) - Strain component, γ_{xy}, for element N.

27th card type is neglected if NREAD = 0 on the 2nd card type, i.e., <u>these cards are only needed to restart the computation</u>.

Repeat 14th through 24th card types for all subsequent construction stages.

TABLE A-1
Manipulation of Scratch Tapes for Storing Stiffness and
Element Strain-Displacement Transformation Matrix

NCT	NNN	NIT	NANALY=0		NANALY=1			
			NSPT =0	NSPT =1	NSPT=0		NSPT=1	
					NYD= 0	NYD> 0	NYD= 0	NYD> 0
1	1	1	NCALC= 1	NCALC= 1	NCALC= 1	NCALC= 1	NCALC= 1	NCALC= 1
		2	4	2	4	3	2	2
		3	4	5	4	6	5	3
		4	4	5	4	6	5	6
	2	1	4	4	4	4	4	4
		2	4	5	4	3	5	5
		3	4	5	4	6	5	3
		4	4	5	4	6	5	6
	3	1	4	4	4	4	4	4
		2	4	5	4	3	5	5
		3	4	5	4	6	5	3
		4	4	5	4	6	5	6
2	1	1	1	1	1	1	1	1
		2	4	2	4	3	2	2
		3	4	5	4	6	5	3
		4	4	5	4	6	5	6
	2	1	4	4	4	4	4	4
		2	4	5	4	3	5	5
		3	4	5	4	6	5	3
		4	4	5	4	6	5	6

NOTE:

NCT = Construction stage number

NNN = Load increment number

NIT = Iteration number

NANALY = 0 Use constant elastic stiffness

NANALY = 1 Use both initial and tangent stiffness during computation

NSPT = 0 No element to be converted to liner

NSPT = 1 Some elements converted to liner

NYD = Number of elements yielded

NCALC = 1, Compute elastic stiffness and store on tapes 80 & 90

NCALC = 2, Compute elastic stiffness with elements converted to liner and store on tapes 50 & 60

NCALC = 3, Compute tangent stiffness and store on tapes 30 & 40

NCALC = 4, Use elastic stiffness on tapes 80 & 90

NCALC = 5, Use elastic stiffness with liner on tapes 50 & 60

NCALC = 6, Use tangent stiffness on tapes 30 & 40

APPENDIX B

COMPUTER PROGRAM LISTING

```
      PROGRAM    NTJTEP2(INPUT,OUTPUT,TAPE9,TAPE90,TAPE91,TAPE3,
     1 PUNCHB,TAPE80,TAPE30,TAPE40,TAPE50,TAPE60,TAPE5=INPUT,
     2 TAPE1=PUNCHB,PUNCH)
C     NO TENSION,JOINT PERTURBATION,AND ELASTIC-PLASTIC ANALYSIS OF
C     PLANE STRAIN STRUCTURES WITH INCREMENTAL CONSTRUCTION.
C**** THIS PROGRAM DEVELOPED BY C-Y CHANG OF WOODWARD-LUNDGREN SEPT. 73
      COMMON /    / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2    ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1        ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /PSLD/ IJBC(50,2),PSCA(75,3),NPBC(75)
      COMMON /BANARG/ B(180),A(90,180),MBAND,ND2,NUMBLK,MBMAX,NB
     1,MTAP1,MTAP2
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      COMMON /JNT/ FN(450),FT(450),NJT
      COMMON /NPS/ PSCAV(75,3),IJBCA(50,2),NS(75)
      DIMENSION DISP(999,2),FY(900),NEJT(450)
      DIMENSION STRN(900,3)
      DIMENSION NE(150),MT(150),NSEL(4,50),TITLE(8),MSP(150)
      EQUIVALENCE (DISP,A(2000)),(FY,A(4000))
      EQUIVALENCE (NEJT,A(6000))
      EQUIVALENCE (STRN,A(6500))
      DATA MBMAX/90/, ND2/180/, END/3HEND/, NB/45/
C     READ AND PRINT OF CONTROL INFORMATION AND MATERIAL PROPERTIES
      LBAD = 0
   50 READ 1000, HED
      IF (HED(1) .EQ. END)  GO TO 9000
C*****FINAL CARD IN DECK MUST READ* END *
      READ 1009, NUMNP,NUMEL,NUMMAT,ACELX,ACELY,NRES,REFPRS,DEPTH,
     1NANALY,NCONST,NPUNCH,NREAD
      PRINT 2000,HED,NUMNP,NUMEL,NUMMAT,ACELX,ACELY
      PRINT 2070, REFPRS,DEPTH,NRES,NANALY,NCONST
      IF (NANALY .EQ. 0) PRINT 3800
      IF (NANALY .EQ. 1) PRINT 3900
C **** NANALY=0 INITIAL STRESS METHOD WITH CONSTANT STIFFNESS
C **** NANALY=1 INITIAL STRESS METHOD WITH CHANGING STIFFNESS AT EACH
C ******INCREMENT OF LOAD
   51 IF (NREAD .EQ. 1) PRINT 3510
      IF (NPUNCH .EQ. 1) PRINT 3520
      READ 1005, MJOINT,MTENS
      IF (MJOINT .EQ. 0) GO TO 52
      READ 1005, (MJNT(I),I=1,MJOINT)
      PRINT 3200,(MJNT(I),I=1,MJOINT)
   52 IF (MTENS .EQ. 0) GO TO 53
      READ 1005, (MNTEN(I),I=1,MTENS)
      PRINT 3300,(MNTEN(I),I=1,MTENS)
   53 CONTINUE
      DO 59 M=1,NUMMAT
      READ 1001,MTYPE,RO(MTYPE),AKO(MTYPE)
C *** CRAC IS DECIMAL FRACTION OF TENSILE STRENGTH APPLIED TO CHECK NO
```

```
C *** TENSION CONVERGENCE
      READ 1030, (E(J,MTYPE),J=1,6),NTEST,CRAC(MTYPE)
      IF (MJOINT .EQ. 0) GO TO 55
      DO 54 I=1,MJOINT
   54 IF (M .EQ. MJNT(I)) GO TO 58
   55 PRINT 2011,MTYPE,RO(MTYPE),AKO(MTYPE),(E(J,MTYPE),J=1,6),NTEST
     1                 ,CRAC(MTYPE)
C*** NTEST=0 IF C AND PHI OBTAINED FROM TRIAXIAL TEST
C*** NTEST=1 IF C AND PHI OBTAINED FROM PLANE STRAIN TEST
      ANG=E(6,MTYPE)/57.29577
      E(6,MTYPE)=ANG
      IF(NTEST.EQ.0) GO TO 56
      TANG=TAN(ANG)
      FD=SQRT(9.+12.*TANG**2)
      E(7,MTYPE)=TANG/FD
      E(8,MTYPE)=3.*E(5,MTYPE)/FD
      GO TO 59
   56 SINA=SIN(ANG)
      COSA=COS(ANG)
      FD=1.732*(3.0-SINA)
      E(7,MTYPE)=2.0*SINA/FD
      E(8,MTYPE)=6.0*E(5,MTYPE)*COSA/FD
      GO TO 59
   58 PRINT 2017, MTYPE, (E(J,MTYPE), J=1,5)
   59 CONTINUE
C     READ AND PRINT OF NODAL POINT DATA
      PRINT    2004
      L = 1
   60 READ      1002, N,CODE(N),R(N),Z(N),UR(N),UZ(N)
      IF (N - L)  100, 90, 70
   70 ZX=N-L+1
      DR = (R(N) - R(L-1)) / ZX
      DZ = (Z(N) - Z(L-1)) / ZX
   80 CODE(L)=0.0
      R(L)=R(L-1)+DR
      Z(L)=Z(L-1)+DZ
      UR(L)=0.0
      UZ(L)=0.0
   90 PRINT 2002, L, CODE(L), R(L), Z(L), UR(L), UZ(L)
      IF (L .EQ. NUMNP)  GO TO 110
      L = L + 1
      IF (N - L)  60, 90, 80
  100 PRINT    2009, N
      LBAD = LBAD + 1
      GO TO 60
  110 CONTINUE
      NCALC=1
C       READ AND PRINT OF ELEMENT PROPERTIES
C     DETERMINE BAND WIDTH
      PRINT    2001
      MBAND = 0
      N = 1
  130 READ 1005,M,(IX(M,I),I=1,5)
      DO 131  I=1,3
      I1 = I + 1
      DO 131  L = I1,4
```

```
      KK = IABS(IX(M,I) - IX(M,L))                                      NTJT113C
      IF (KK .GE. NB)  GO TO 179                                        NTJT114C
  131 IF (KK .GT. MBAND)  MBAND = KK                                    NTJT115C
      IF (M - N)  180, 170, 150                                         NTJT116C
  150 IX(N,1)=IX(N-1,1)+1                                               NTJT117C
      IX(N,2)=IX(N-1,2)+1                                               NTJT118C
      IX(N,3)=IX(N-1,3)+1                                               NTJT119C
      IX(N,4)=IX(N-1,4)+1                                               NTJT120C
      IX(N,5)=IX(N-1,5)                                                 NTJT121C
  170 PRINT     2003, N,(IX(N,I),I=1,5)                                 NTJT122C
      IF (N .EQ. NUMEL)  GO TO 190                                      NTJT123C
      N = N + 1                                                         NTJT124C
      IF (M - N)  130, 170, 150                                         NTJT125C
  179 N = N + 1                                                         NTJT126C
  180 PRINT 2018, M, KK, N                                              NTJT127C
      LBAD = LBAD + 1                                                   NTJT128C
      GO TO 130                                                         NTJT129C
  190 CONTINUE                                                          NTJT130C
      MBAND = 2 * MBAND + 2                                             NTJT131C
      PRINT 2012,MBAND                                                  NTJT132C
C***********************************************************************NTJT133C
C     READ INITIAL STRESSES FOR THE PROBLEM, PRINT AS PART OF FIRST STEPNTJT134C
C **********************************************************************NTJT135C
      DO 32 N=1,NUMEL                                                   NTJT136C
      STRS(N,4)=0.0                                                     NTJT137C
      DO 32 I=1,3                                                       NTJT138C
      SEP(N,I)=0.0                                                      NTJT139C
   32 STRS(N,I)=0.0                                                     NTJT140C
      IF (NRES .LE. 0) GO TO 44                                         NTJT141C
      IF(NRES  .EQ. 3) GO TO 48                                         NTJT142C
      L=1                                                               NTJT143C
C*NRES=-1 RESIDUAL STRESS ARE GENERATED FROM WHICH RESIDUAL LOAD CALCULANTJT144C
C*NRES=0  RESIDUAL STRESS ARE GENERATED,BUT RESIDUAL LOAD IS ZERO       NTJT145C
C*NRES=1  RESIDUAL STRESS ARE INPUT AS DATA FROM WHICH RESIDUAL LOAD GENNTJT146C
C*NRES=2  RESIDUAL STRESS ARE INPUT AS DATA,BUT RESIDUAL LOAD IS ZERO   NTJT147C
C*NRES=3 COMPUTE RESIDUAL STRESSES BEFORE ANALYSIS FOR COMPUTING FORCES NTJT148C
C*******  ON EXCAVATED FACE*********                                    NTJT149C
   47 READ 1007, N, (STRS(N,I),I=1,4)                                   NTJT150C
      IF(N-L) 40,41,42                                                  NTJT151C
   42 DO 46 I=1,4                                                       NTJT152C
   46 STRS(L,I)=STRS(L-1,I)                                             NTJT153C
   41 IF (L .EQ. NUMEL)  GO TO 45                                       NTJT154C
      L = L + 1                                                         NTJT155C
      IF (N - L)  47, 41, 42                                            NTJT156C
   40 LBAD=LBAD+1                                                       NTJT157C
      PRINT 1008,N,LBAD,L                                               NTJT158C
      GO TO 47                                                          NTJT159C
   45 CONTINUE                                                          NTJT160C
      PRINT 1006                                                        NTJT161C
      PRINT 1007, (K,(STRS(K,I),I=1,4),K=1,NUMEL)                       NTJT162C
      IF (MJOINT .EQ. 0) GO TO 44                                       NTJT163C
      READ 4100,NJT,(FN(I),FT(I),NEJT(I),I=1,NJT)                       NTJT164C
      PRINT 4200,(I,FN(I),FT(I),NEJT(I),I=1,NJT)                        NTJT165C
      GO TO 44                                                          NTJT166C
   48 DO 210 N = 1, NUMEL                                               NTJT167C
      MTYPE=IX(N,5)                                                     NTJT168C
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      I=IX(N,1)                                                         NTJT169C
      J=IX(N,2)                                                         NTJT170C
       K=IX(N,3)                                                        NTJT171C
      L=IX(N,4)                                                         NTJT172C
      IF (K .EQ. L) GO TO 204                                           NTJT173C
      ZZZ(5)=(Z(I)+Z(J)+Z(K)+Z(L))*0.25                                 NTJT174C
      GO TO 205                                                         NTJT175C
  204 ZZZ(5)=(Z(I)+Z(J)+Z(K))/3.                                        NTJT176C
  205 IF(RO(MTYPE).EQ.0. .AND. E(2,MTYPE) .LE. 1.)GO TO 210             NTJT177C
      CALL INITST                                                       NTJT178C
  210 CONTINUE                                                          NTJT179C
      NRES=0                                                            NTJT180C
   44 CONTINUE                                                          NTJT181C
C     SOLVE NON-LINEAR STRUCTURE BY SUCCESSIVE APPROXIMATIONS           NTJT182C
      DO 350 N=1,NUMEL                                                  NTJT183C
      EPS(N)=0.0                                                        NTJT184C
      MTAG(N)=1                                                         NTJT185C
  350 CONTINUE                                                          NTJT186C
      READ  1005,NCPSNT,NPRSNT,ITPSNT                                   NTJT187C
      PRINT 6000,NCPSNT,NPRSNT,ITPSNT                                   NTJT188C
      IT1=ITPSNT                                                        NTJT189C
      NPRS1=NPRSNT                                                      NTJT190C
      NELPNCH=0                                                         NTJT191C
      DO 900 NCT=NCPSNT,NCONST                                          NTJT192C
      IF(NCT.GT.1) NRES=0                                               NTJT193C
      IF(NCT.GT.NCPSNT) NPRSNT=1                                        NTJT194C
      READ 1000, TITLE                                                  NTJT195C
      READ 1005,NEPSP,NUMPC,NPF,NPCAV,NP,NGLD,MTRM,NSPT                 NTJT196C
      IF(NCT.EQ.NCPSNT) NP1=NP                                          NTJT197C
      PRINT 6001,TITLE,NERSP,NUMPC,NPF,NPCAV,NP,MTRM,NSPT               NTJT198C
C** NSPT = 1 IF THERE IS LINER FOR THIS CONSTRUCTION STEP               NTJT199C
C** NSPT = 0 IF THERE IS NO LINER FOR THIS CONSTRUCTION STEP            NTJT200C
      IF (NGLD .EQ. 1) PRINT 6200                                       NTJT201C
      IF(NCT.GT.NCPSNT) GO TO 610                                       NTJT202C
      IF(NPRSNT.LE.NP) GO TO 610                                        NTJT203C
      PRINT 3600                                                        NTJT204C
      GO TO 9000                                                        NTJT205C
  610 CONTINUE                                                          NTJT206C
      IF(NERSP.LE.0) GO TO 810                                          NTJT207C
      NELPNCH=NELPNCH+1                                                 NTJT208C
      NCALC=1                                                           NTJT209C
      READ 1005,(NE(I),MT(I),MSP(I),I=1,NERSP)                          NTJT210C
      PRINT 6002                                                        NTJT211C
      PRINT6007,(NE(I),MT(I),MSP(I),I=1,NERSP)                          NTJT212C
C * MT(I) = MATERIAL NO. ASSIGNED FOR NIT = 1                           NTJT213C
C * MSP(I) = MATERIAL NO. ASSIGNED FOR NIT GREATER THAN 1               NTJT214C
  810 IF(NUMPC.EQ.0) GO TO 820                                          NTJT215C
      PRINT 2005                                                        NTJT216C
      READ 1005,((IJBC(L,I),I=1,2),L=1,NUMPC)                           NTJT217C
      PRINT 2007,((IJBC(L,I),I=1,2),L=1,NUMPC)                          NTJT218C
      READ 1005, NPBCP                                                  NTJT219C
      PRINT 2050                                                        NTJT220C
      DO 815 M=1,NPBCP                                                  NTJT221C
      READ 1020,NPBC(M),(PSCA(M,I),I=1,3)                               NTJT222C
      PRINT 1020 ,NPBC(M),(PSCA(M,I),I=1,3)                             NTJT223C
  815 CONTINUE                                                          NTJT224C
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  820 CONTINUE                                                          NTJT225C
      IF(NPF.EQ.0) GO TO 840                                            NTJT226C
      PRINT 6005                                                        NTJT227C
      DO 830 I=1,NPF                                                    NTJT228C
      READ 1002 ,  N,CODE(N),UR(N),UZ(N)                                NTJT229C
      PRINT 6008,  N,CODE(N),UR(N),UZ(N)                                NTJT230C
  830 CONTINUE                                                          NTJT231C
  840 CONTINUE                                                          NTJT232C
      IF(NPCAV.EQ.0) GO TO 850                                          NTJT233C
      PRINT 6006                                                        NTJT234C
      READ 1010,   (NS(I),(NSEL(J,I),J=1,4),I=1,NPCAV)                  NTJT235C
      PRINT 4000,  (NS(I),(NSEL(J,I),J=1,4),I=1,NPCAV)                  NTJT236C
      READ 1005,NCAVPC                                                  NTJT237C
      READ 1005 , ((IJBCA(L,I),I=1,2),L=1,NCAVPC)                       NTJT238C
      PRINT 2005                                                        NTJT239C
      PRINT 2007 ,((IJBCA(L,I),I=1,2),L=1,NCAVPC)                       NTJT240C
  850 CONTINUE                                                          NTJT241C
C***  IF THE PRESENT CONSTRUCTION STAGE IS NOT THE FIRST ANALYSIS OF     NTJT242C
C     THIS RUN NPRSNT=1                                                 NTJT243C
      READ 1040 ,   (ITN(N),PRATIO(N),N=NPRSNT,NP)                      NTJT244C
C** ITN .GE.2IF NSP = 1                                                 NTJT245C
      PRINT 3400,((N,ITN(N),PRATIO(N)),N=NPRSNT,NP)                     NTJT246C
      IF (NCT .GT. NCPSNT .OR. NREAD .NE. 1 ) GO TO 860                 NTJT247C
      READ (5) ((STRS(N,I),SEP(N,I),I=1,3),STRS(N,4),MTAG(N),N=1,NUMEL) NTJT248C
      IF (MJOINT .EQ. 0) GO TO 860                                      NTJT249C
      READ (5) NJT,(FN(N),FT(N),N=1,NJT)                                NTJT250C
  860 IF(NPCAV .EQ. 0) GO TO 861                                        NTJT251C
      CALL NPSTRS(NSEL,NPCAV)                                           NTJT252C
  861 CONTINUE                                                          NTJT253C
      IF( NERSP .LE. 0) GO TO 870                                       NTJT254C
      DO 865 I=1,NERSP                                                  NTJT255C
      N=NE(I)                                                           NTJT256C
      MTAG(N)=1                                                         NTJT257C
      IF (MTRM .EQ. MT(I)) GO TO 863                                    NTJT258C
      GO TO 865                                                         NTJT259C
  863 MTAG(N)=0                                                         NTJT260C
      DO 864 J=1,4                                                      NTJT261C
      STRS(N,J)=0.0                                                     NTJT262C
  864 CONTINUE                                                          NTJT263C
  865 CONTINUE                                                          NTJT264C
  870 CONTINUE                                                          NTJT265C
      DO 500 NNN=NPRSNT,NP                                              NTJT266C
      ITAPE=0                                                           NTJT
      NISTOP=0                                                          NTJT267C
      ITNP=ITN(NNN)                                                     NTJT268C
      IF(NNN.GT.NPRSNT.OR.NCT.GT.NCPSNT) ITPSNT=1                       NTJT269C
      DO 450 NIT=ITPSNT,ITNP                                            NTJT270C
      NYD  =  0                                                         NTJT271C
      DO 872  N = 1,NUMEL                                               NTJT272C
      IF( MTAG(N) .GE. 4) NYD = NYD + 1                                 NTJT273C
  872 CONTINUE                                                          NTJT274C
      IF(NIT .GE. 3) GO TO 890                                          NTJT275C
      IF (NERSP .EQ. 0) GO TO 890                                       NTJT276C
      IF (NSPT .EQ. 0) GO TO 875                                        NTJT277C
      IF (NIT .EQ. 1) GO TO 875                                         NTJT278C
      IF (NIT .EQ. 2) GO TO 880                                         NTJT279C
      GO TO 890                                                         NTJT280C
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875 DO 877 I=1,NERSP                                                NTJT281C
    N = NE(I)                                                       NTJT282C
877 IX(N,5) = MT(I)                                                 NTJT283C
    GO TO 890                                                       NTJT284C
880 DO 881 I=1,NERSP                                                NTJT285C
    N = NE(I)                                                       NTJT286C
    IF( MSP(I) .NE. MTRM)  MTAG(N)  =  1                            NTJT287C
881 IX(N,5) = MSP(I)                                                NTJT288C
890 CONTINUE                                                        NTJT289C
    IF (NNN .EQ. 1 .AND. NIT .EQ. 1) NCALC = 1                      NTJT290C
    IF (NANALY .EQ. 0 .AND. NSPT .EQ. 0) GO TO 200                  NTJT291C
    IF (NANALY .EQ. 0 .AND. NSPT .EQ. 1) GO TO 220                  NTJT292C
    IF (NANALY .EQ. 1 .AND. NSPT .EQ. 0) GO TO 240                  NTJT293C
    IF (NANALY .EQ. 1 .AND. NSPT .EQ. 1) GO TO 260                  NTJT294C
    CALL EXIT                                                       NTJT295C
200 IF (NNN .EQ. NPRSNT .AND. NIT .EQ. ITPSNT) NCALC = 1            NTJT296C
    IF (NNN .NE. NPRSNT .OR. NIT .NE. ITPSNT) NCALC = 4             NTJT297C
    GO TO 270                                                       NTJT298C
220 IF (NNN .EQ. 1 .AND. NIT .EQ. 2) NCALC = 2                      NTJT299C
    IF (NNN .EQ. 1 .AND. NIT .GT. 2) NCALC = 5                      NTJT300C
    IF (NNN .GT. 1 .AND. NIT .EQ. 1) NCALC = 4                      NTJT301C
    IF (NNN .GT. 1 .AND. NIT .GT. 1) NCALC = 5                      NTJT302C
    GO TO 270                                                       NTJT303C
240 IF (NIT .GE. 2) NCALC=3                                         NTJT304C
    IF (NIT .GT. 2 .AND. ITAPE .EQ. 1) NCALC=6                      NTJT306C
    IF( NYD .EQ. 0 .AND. NIT .GE. 2 ) NCALC = 4                     NTJT306C
    IF (NNN .GT. 1 .AND. NIT .EQ. 1) NCALC = 4                      NTJT307C
    GO TO 270                                                       NTJT308C
260 IF (NNN .EQ. 1 .AND. NIT .EQ. 2) NCALC = 2                      NTJT309C
    IF (NIT .GE. 3) NCALC=3                                         NTJT310C
    IF (NIT .GT. 3 .AND. ITAPE .EQ.1) NCALC=6                       NTJT311C
    IF( NIT .GE. 3 .AND. NYD .EQ. 0 ) NCALC = 5                     NTJT312C
    IF (NNN .GT. 1 .AND. NIT .EQ. 1) NCALC = 4                      NTJT313C
    IF (NNN .GT. 1 .AND. NIT .EQ. 2) NCALC = 5                      NTJT314C
270 CONTINUE                                                        NTJT315C
    MTAP1 = 80                                                      NTJT316C
    MTAP2 = 90                                                      NTJT317C
    IF (NCALC .EQ. 2 .OR. NCALC .EQ. 5) GO TO 280                   NTJT318C
    IF( NCALC .EQ. 3 .OR. NCALC .EQ. 6) GO TO 285                   NTJT319C
    GO TO 290                                                       NTJT320C
280 MTAP1 = 50                                                      NTJT321C
    MTAP2 = 60                                                      NTJT322C
    GO TO 290                                                       NTJT323C
285 MTAP1 = 30                                                      NTJT324C
    MTAP2 = 40                                                      NTJT325C
    ITAPE=1                                                         NTJT
290 CONTINUE                                                        NTJT326C
    PRINT 3100, NNN,NIT                                             NTJT327C
    CALL SECOND(T1)                                                 NTJT328C
    IF (NIT .EQ. 1 .OR. NCALC .EQ. 1) GO TO 357                     NTJT329C
    IF(NCT.GT.NCPSNT) GO TO 760                                     NTJT330C
    IF(NNN.EQ.NPRSNT.AND.NIT.EQ.IT1) GO TO 357                      NTJT331C
760 IF(NPF.GT.0) GO TO 770                                          NTJT332C
    IF (NNN .EQ. NPRSNT .AND. NIT .EQ. IT1) GO TO 357               NTJT333C
    IF (MJOINT .EQ. 0) GO TO 355                                    NTJT334C
770 DO 354 N=1,NUMNP                                                NTJT335C
    IF (ABS(UR(N)) .LE. 1.) GO TO 354                               NTJT336C
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      IF (ABS(UZ(N)) .LE. 1.) GO TO 354                                 NTJT337C
      NISTOP=NISTOP+1                                                   NTJT338C
C*****                                                                  NTJT339C
C*****                                                                  NTJT340C
  354 CONTINUE                                                          NTJT341C
  355 CONTINUE                                                          NTJT342C
      IF (NNN.EQ.1.AND.NSPT.EQ.1.AND.NIT.EQ.2.AND.NISTOP.GT.0) GO TO 357NTJT343C
      IF (NISTOP .EQ. 0) GO TO 460                                      NTJT344C
  357 CONTINUE                                                          NTJT344C
C     FORM STIFFNESS MATRIX                                             NTJT345C
      CALL STIFF                                                        NTJT346C
      CALL SECOND(T2)                                                   NTJT347C
      IF (LBAD .NE. 0)  GO TO 8950                                      NTJT348C
C     SOLVE FOR DISPLACEMENTS                                           NTJT349C
      CALL BANSOL (NNN,NIT,NCALC)                                       NTJT350C
C                                                                       NTJT351C
      CALL SECOND(T3)                                                   NTJT352C
      PRINT 2016                                                        NTJT353C
      DO 360 N=1,NUMNP                                                  NTJT354C
      DO 360 I=1,2                                                      NTJT354C
  360 DISP(N,I)=0.0                                                     NTJT356C
      DO 361 N=1,NUMEL                                                  NTJT357C
      DO 362 I=1,3                                                      NTJT357C
  362 STRN(N,I)=0.0                                                     NTJT359C
      FY(N)=-1000.                                                      NTJT360C
  361 CONTINUE                                                          NTJT361C
      IF(NNN .EQ. 1 .AND. NIT .EQ. 1.AND.NCT.EQ.1) GO TO 380            NTJT362C
      IF(NREAD.EQ.1.AND.NNN.EQ.NPRSNT.AND.NIT.EQ.IT1) GO TO 370         NTJT363C
      REWIND 3                                                          NTJT364C
      READ (3) ((DISP(N,I),I=1,2),N=1,NUMNP),(FY(N),(STRN(N,I),I=1,3),  NTJT365C
     1N=1,NUMEL)                                                        NTJT366C
      GO TO 380                                                         NTJT367C
  370 READ (5) ((DISP(N,I),I=1,2),N=1,NUMNP),(FY(N),(STRN(N,I),I=1,3),  NTJT368C
     1N=1,NUMEL)                                                        NTJT369C
  380 DO 400 N=1,NUMNP                                                  NTJT370C
      N2=N*2                                                            NTJT371C
      DISP(N,1)=DISP(N,1)+B(N2-1)                                       NTJT372C
      DISP(N,2)=DISP(N,2)+B(N2)                                         NTJT373C
      PRINT 2006, N,B(N2-1),B(N2),CODE(N),DISP(N,1),DISP(N,2)           NTJT374C
  400 CONTINUE                                                          NTJT375C
      IF(NGLD.NE.1) GO TO 410                                           NTJT376C
      DO 405 N=1,NUMNP                                                  NTJT377C
      DISP(N,1)=0.0                                                     NTJT378C
      DISP(N,2)=0.0                                                     NTJT379C
  405 CONTINUE                                                          NTJT380C
  410 CONTINUE                                                          NTJT381C
C     COMPUTE STRESSES                                                  NTJT382C
      NISTOP=0                                                          NTJT383C
      CALL STRESS                                                       NTJT384C
C     RESET UR AND UZ EQUAL TO ZERO                                     NTJT385C
      IF( NIT .GE. 2) GO TO 415                                         NTJT386C
      IF(NERSP.EQ. 0) GO TO 415                                         NTJT387C
      DO 414  I = 1,NERSP                                               NTJT388C
      IF( MTRM  .NE.  MT(I)) GO TO 414                                  NTJT389C
      N  =  NE(I)                                                       NTJT390C
      DO 413 J = 1,3                                                    NTJT391C
                                                                        NTJT392C
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      STRN(N,J)  = 0.
  413 CONTINUE                                                          NTJT393C
  414 CONTINUE                                                          NTJT394C
  415 CONTINUE                                                          NTJT395C
      DO 420 N=1,NUMNP                                                  NTJT396C
      UR(N)=0.                                                          NTJT397C
      UZ(N)=0.                                                          NTJT398C
  420 CONTINUE                                                          NTJT399C
      IF(MJOINT.EQ.0) GO TO 409                                         NTJT400C
      CALL JTSTR                                                        NTJT401C
  409 CONTINUE                                                          NTJT402C
      REWIND 3                                                          NTJT403C
      WRITE(3) ((DISP(N,I),I=1,2),N=1,NUMNP),(FY(N),(STRN(N,I),I=1,3),  NTJT404C
     1N=1,NUMEL)                                                        NTJT405C
      CALL SECOND(T4)                                                   NTJT406C
      TT=T4-T1                                                          NTJT407C
      T1=T2-T1                                                          NTJT408C
      T2=T3-T2                                                          NTJT409C
      PRINT 2020,T1,T2,TT                                               NTJT410C
      NREAD=0                                                           NTJT411C
  450 CONTINUE                                                          NTJT412C
  460 CONTINUE                                                          NTJT413C
  500 CONTINUE                                                          NTJT414C
  900 CONTINUE                                                          NTJT415C
C****PUNCH OUT INITIAL STRESSES FROM GRAVITY TURN-ON ANALYSIS            NTJT416C
      IF (NGLD .NE. 1 .OR.  NRES .NE. -1) GO TO 510                     NTJT417C
      PUNCH 1000,HED                                                    NTJT418C
      PRINT 1007,(N,(STRS(N,I),I=1,4),N=1,NUMEL)                        NTJT419C
      PUNCH 4300 ,(N,(STRS(N,I),I=1,4),N=1,NUMEL)                       NTJT420C
  510 CONTINUE                                                          NTJT421C
      IF (NPUNCH .NE. 1) GO TO 600                                      NTJT422C
      PUNCH 1000,HED                                                    NTJT423C
      PUNCH 3700,NCONST,NP                                              NTJT424C
      IF(NELPNCH.LE.0) GO TO 520                                        NTJT425C
      PUNCH 6100,NELPNCH                                                NTJT426C
      PUNCH 1015,((M,(IX(M,I),I=1,5)),M=1,NUMEL)                        NTJT427C
  520 WRITE(1) ((STRS(N,I),SEP(N,I),I=1,3),STRS(N,4),MTAG(N),N=1,NUMEL) NTJT428C
      IF (MJOINT .EQ. 0) GO TO 550                                      NTJT429C
      WRITE (1) NJT,(FN(N),FT(N),N=1,NJT)                               NTJT430C
  550 CONTINUE                                                          NTJT431C
      WRITE(1) ((DISP(N,I),I=1,2),N=1,NUMNP),(FY(N),(STRN(N,I),I=1,3),  NTJT432C
     1N=1,NUMEL)                                                        NTJT433C
  600 CONTINUE                                                          NTJT434C
      GO TO 50                                                          NTJT435C
 1000 FORMAT (8A10)                                                     NTJT436C
 1001 FORMAT (I5,2F10.5)                                                NTJT437C
 1002 FORMAT (I5,F5.0,5F10.0)                                           NTJT438C
 1004 FORMAT (2I5,F10.0)                                                NTJT439C
 1005 FORMAT (16I5)                                                     NTJT440C
 1006 FORMAT (37H1      I N I T I A L   S T R E S S E S/8H0ELEMENT3X,7HXSNTJT441C
     1TRESS8X,7HYSTRESS7X,8HXYSTRESS)                                   NTJT442C
 1007 FORMAT (I5,4E15.5)                                                NTJT443C
 1008 FORMAT(* RESIDUAL STRESS INPUT ERROR,N=*,I10,*LBAD=*,I10,*L=*,I10)NTJT444C
 1009 FORMAT (3I5,2F10.2,I5,2F10.5,4I5)                                 NTJT445C
 1010 FORMAT(5I5)                                                       NTJT446C
 1015 FORMAT(6I5)                                                       NTJT447C
                                                                        NTJT448C
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 1020 FORMAT (I5,3F10.0)                                                 NTJT449C
 1030 FORMAT(6F10.5,I5,F10.5)                                            NTJT450C
 1040 FORMAT (I5,F10.5)                                                  NTJT451C
 2000 FORMAT (1H120X,8A10/                                               NTJT452C
     1 30H0 NUMBER OF NODAL POINTS------ I3 /                            NTJT453C
     2 30H0 NUMBER OF ELEMENTS---------- I3 /                            NTJT454C
     3 30H0 NUMBER OF DIFF. MATERIALS--- I3 /                            NTJT455C
     5 30H0 X-ACCELERATION-------------- E12.4/                          NTJT456C
     6 30H0 Y-ACCELERATION-------------- E12.4/  )                       NTJT457C
 2001 FORMAT (49H1ELEMENT NO.      I     J     K     L    MATERIAL  )    NTJT458C
 2002 FORMAT (I12,F12.2,2F12.3,2E24.7)                                   NTJT459C
 2003 FORMAT (1I13,4I6,1I12)                                             NTJT460C
 2004 FORMAT (108H1NODAL POINT        TYPE   X-ORDINATE  Y-ORDINATE  X LONTJT461C
     1AD OR DISPLACEMENT  Y LOAD OR DISPLACEMENT              )          NTJT462C
 2005 FORMAT (29H0PRESSURE BOUNDARY CONDITIONS/ 12H     I     J //)      NTJT463C
 2006 FORMAT (I12,1P2E20.7,0PF20.0,1P2E20.7)                             NTJT464C
 2016 FORMAT (12H1N.P. NUMBER18X,2HUX18X,2HUY16X,4HCODE12X,8HUX CUMUL12XNTJT465C
     1,8HUY CUMUL)                                                       NTJT466C
 2007 FORMAT (2I6)                                                       NTJT467C
 2009 FORMAT (26H0NODAL POINT CARD ERROR N= I5)                          NTJT468C
 2011 FORMAT (15H0MATERIAL NO. =I3,16H  MASS DENSITY =E12.4,41H  RATIO ONTJT469C
     1F HORIZONTAL TO VERTICAL STRESS= F10.5/1X,16HTENSILE STRENGTH,1X,  NTJT470C
     2 12HCOMP MODULUS,2X,13HPOISSON RATIO,2X,12HTENS MODULUS,3X, 8HCOHENTJT471C
     3SION,7X,17HANGLE OF FRICTION,5X,14HTYPE OF TEST= /6E15.5,I10/      NTJT472C
     4 10X,56HFRACTION OF TENSILE STRENGTH FOR NO TENSION CONVERGENCE=,  NTJT473C
     5 F10.5//)                                                          NTJT474C
 2012 FORMAT (/6HMBAND= I5 /)                                            NTJT475C
 2017 FORMAT (1H010X,15HMATERIAL NUMBERI5/1H06X,2HKN13X,2HKT14X,1HC13X,3NTJT476C
     1HPHI8X,12HMAX. CLOSURE/5E15.4)                                     NTJT477C
 2018 FORMAT (16H ELEMENT CARD NOI6,5X,4HKK =I4,5X,8HNEXT N =I6)         NTJT478C
 2019 FORMAT (23H NUMBER OF ERRORS FOUNDI4)                              NTJT479C
 2020 FORMAT (13H0TIMING-STIFFF8.3,5X,6HBANSOLF8.3,5X,9HITERATIONF8.3)   NTJT480C
 2030 FORMAT (/*0PLANE STRAIN ANALYSIS OF JOINTED STRUCTURES*)           NTJT481C
 2050 FORMAT (26H INITIAL STRESSES AT NODES /35H N.P.     SIGXX     SIGYNTJT482C
     1Y     SIGXY )                                                      NTJT483C
 2070 FORMAT (5X,41HVERTICAL STRESS AT REFERENCE POINT (PSF)= F20.2/     NTJT484C
     15X,34HELEVATION AT REFERENCE POINT (FT)= F20.2/5X,5HNRES= ,I10 ,   NTJT485C
     2 17HTYPE OF ANALYSIS= ,I5/5X,27HNO. OF CONSTRUCTION STEPS= ,I5//)  NTJT486C
 3002 FORMAT (10F7.4)                                                    NTJT487C
 3100 FORMAT (//18H0NO. OF INCREMENT= ,I5,5X,17HNO. OF ITERATION= ,I5/)  NTJT488C
 3200 FORMAT (29H MAT. NO. FOR JOINT ELEMENTS= ,12I6)                    NTJT489C
 3300 FORMAT (36H MAT. NO. WHICH CAN SUSTAIN TENSION= ,12I6)             NTJT490C
 3400 FORMAT (21H LOADING INCREMENT N= ,I5,5X,38HNO. OF ITERATIONS FOR TNTJT491C
     1HIS INCREMENT = ,I5,5X,31HPERCENTAGE OF PRESSURE APPLIED=, F10.5)  NTJT492C
 3500 FORMAT (//31H PRESENT LOADING INCREMENT NO.=,I5,5X,14HITERATION NONTJT493C
     1.=,I5/ 7H NREAD=,I5/ 8H NSTSRT=,I5,5X,63HNSTSRT.NE.0 STRESSES IN RNTJT494C
     2-T DIRECTIONS WILL ALSO BE PRINTED OUT /)                          NTJT495C
 3510 FORMAT(/5H ****,42HDATA FROM LAST INCREMENT ARE READ AS INPUT )    NTJT496C
 3520 FORMAT(/5H ****,64HDATA WILL BE PUNCHED OUT AT THE LAST ITERATION  NTJT497C
     1OF LAST INCREMENT )                                                NTJT498C
 3600 FORMAT(74H PRESENT LOADING INCREMENT NO. IS GREATER THAN LAST INCRNTJT499C
     1EMENT NO. **STOP** )                                               NTJT500C
 3700 FORMAT (55H THE FOLLOWING DATA ARE FINAL AND EXCESS STRESS,NCONST=NTJT501C
     1  ,I5, 3HNP= , I5)                                                 NTJT502C
 3800 FORMAT (* USING INITIAL STRESS METHOD WITH CONSTANT STIFFNESS*)    NTJT503C
 3900 FORMAT (* USING INITIAL STRESS METHOD WITH INITIAL STIFFNESS AT EANTJT504C
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     1CH INCREMENT OF LOAD BUT CHANGING STIFFNESS IN SUBSEQUENT ITERATIONTJT505C
     2NS*)                                                              NTJT506C
 4000 FORMAT(//* N.P. ALONG EXCAVATED FACE SURROUNDING ELEMENTS * /     NTJT507C
     * (I20,10X,4I5))                                                   NTJT508C
 4100 FORMAT(I5/(2F20.5,I5))                                            NTJT509C
 4200 FORMAT (///* INITIAL NORMAL AND TANGENTIAL STRESSES FOR JOINTS*// NTJT510C
     1 * JOINT NO.  NORMAL STRESS    TANGENTAL STRESS    EL. NO. */(   NTJT511C
     2  I10,2E15.5,I10))                                                NTJT512C
 4300 FORMAT (* INITIAL STRESSES*/(I5,4E15.5))                          NTJT513C
 6000 FORMAT(                                                           NTJT514C
     1 50H0 NUMBER OF PRESENT CONST. STEP-------------------    I5/     NTJT515C
     2 50H0 NUMBER OF PRESENT LOAD INCREMENT----------------    I5/     NTJT516C
     3 50H0 NUMBER OF PRESENT ITERATION -------------------    I5/   ) NTJT517C
 6001 FORMAT(1H1,8A10//                                                 NTJT518C
     1 5X,* FOLLOWING DATA ARE REQUIRED FOR PRESENT CONSTRUCTION STAGE* NTJT519C
     2 //5X,*NUMBER OF ELEMENTS TO BE EXCAVATED AND OR FOR STRUCTURAL   NTJT520C
     3SUPPORT = *, I10,/5X,* NUMBER OF PRESSURE CARDS = *,I10,/5X, *    NTJT521C
     4NUMBER OF NODAL POINTS AT WHICH FORCES ARE APPLIED *,I10,/ 5X,    NTJT522C
     5*NUMBER OF NODAL POINTS ALONG CURRENT EXCAVATED FACE REQIRED FOR  NTJT523C
     6 EXCAVATION SIMULATION = *,I10,/5X,* NUMBER OF LOAD INCREMENTS = *NTJT524C
     7,I10/ 5X,* MATERIAL TYPE FOR EXCAVATED ELEMENTS= *,I10/           NTJT525C
     8 5X,* INDEX(=1 IF THERE IS LINER) FOR LINER = *, I10//)           NTJT526C
 6002 FORMAT( // 5X,* MATERIAL TYPE FOR FOLLOWING ELEMENTS ARE CHANGED  NTJT527C
     1 AS INDICATED *//5X, * ELEMENT NO.*,5X,                           NTJT528C
     2 *MAT. TYPE(NIT=1)*, 2X, *MAT. TYPE(NIT GT 1)*)                   NTJT529C
 6005 FORMAT( //5X,*FOLLOWING BOUNDARY CONDITIONS ARE CHANGED FOR CURRENNTJT530C
     1T CONST. STAGE *,//1X,* NODAL POINT*,2X,* TYPE*,2X,* X LOAD OR DISNTJT531C
     2PLACEMENT*,2X,* Y LOAD OR DISPLACEMENT*)                          NTJT532C
 6006 FORMAT(// 5X,* PRESSURE BOUNDARY CONDITIONS TO SIMULATE EXCAVATIONNTJT533C
     1  *                    )                                          NTJT534C
 6007 FORMAT (2I12,I20)                                                 NTJT535C
 6008 FORMAT(I8,F9.2,2E30.5)                                            NTJT536C
 6100 FORMAT( * ELEMENT PROPERTY HAS BEEN CHANGED  *,I5, 2X,* TIMES*)   NTJT537C
 6200 FORMAT(// 5X, * THE PRESENT STEP IS GRAVITY TURN ON TO OBTAIN     NTJT538C
     * INITIAL STRESSES * //)                                           NTJT539C
C                                                                       NTJT540C
 8950 PRINT 2019, LBAD                                                  NTJT541C
 9000 CONTINUE                                                          NTJT542C
      END                                                               NTJT543C
```

```
      SUBROUTINE NPSTRS(NSEL,NPST)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /NPS/ PSCAV(75,3),IJBCA(50,2),NS(75)
      DIMENSION C(4,4),F(3,4),NSEL(4,50)
      PRINT 1005
      DO 500 I=1,NPST
      NC=NS(I)
      DO 200 J=1,4
      MM=NSEL(J,I)
      IN=IX(MM,1)
      JN=IX(MM,2)
      KN=IX(MM,3)
      LN=IX(MM,4)
      XX=(R(IN)+R(JN)+R(KN)+R(LN))*0.25
      YY=(Z(IN)+Z(JN)+Z(KN)+Z(LN))*0.25
      C(J,1)=1.
      C(J,2)=XX
      C(J,3)=YY
  200 C(J,4)=XX*YY
      DO 380 N=1,4
      D=C(N,N)
      DO 330 J=1,4
  330 C(N,J)=-C(N,J)/D
      DO 370 K=1,4
      IF (N .EQ. K) GO TO 370
      DO 360 J=1,4
      IF (N .EQ. J) GO TO 360
      C(K,J)=C(K,J)+C(K,N)*C(N,J)
  360 CONTINUE
  370 C(K,N)=C(K,N)/D
      C(N,N)=1./D
  380 CONTINUE
      DO 390 K=1,3
      DO 390 L=1,4
  390 F(K,L)=0.0
      DO 450 K=1,3
      DO 450 L=1,4
      DO 450 M=1,4
      MM=NSEL(M,I)
      F(K,L)=F(K,L)+C(L,M)*STRS(MM,K)
  450 CONTINUE
      DO 460 K=1,3
  460 PSCAV(I,K)=F(K,1)+F(K,2)*R(NC)+F(K,3)*Z(NC)+F(K,4)*R(NC)*Z(NC)
      PRINT 1007,NC,(PSCAV(I,K),K=1,3)
  500 CONTINUE
      RETURN
 1005 FORMAT (1H1,/* N.P.        SIGXX        SIGYY        SIGXY*//)
 1007 FORMAT (I5,3E15.5)
      END
```

```
      SUBROUTINE STIFF
C**** CALCULATION OF STIFFNESS MATRIX FOR FIRST STEP ONLYIFNANALY=0
C**** CALCULATION OF B ARRAY FOR EACH TIME STEP
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2    ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1        ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /PSLD/ IJBC(50,2),PSCA(75,3),NPBC(75)
      COMMON /NPS/ PSCAV(75,3),IJBCA(50,2),NS(75)
      COMMON /BANARG/ B(90),B2(90),A(90,90),A2(90,90),MBAND,ND2,NUMBLK,
     1                MBMAX,NB,MTAP1,MTAP2
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      COMMON /JNT/ FN(450),FT(450),NJT
      ND=MBMAX
C     INITIALIZATION
      NISTOP=0
      PRINT 2010
      REWIND 9
      IF (MTAP1 .EQ. 80) PRINT 3000, NCALC
      IF (MTAP1 .EQ. 30) PRINT 3010, NCALC
      IF (MTAP1 .EQ. 50) PRINT 3020, NCALC
 3000 FORMAT(// 10X,* THIS ITERATION USES ELASTIC STIFFNESS WITHOUT LINE
     1R *, 5X, * NCALC=*, I5//)
 3010 FORMAT(// 10X,* THIS ITERATION USES TANGENT  STIFFNESS *,5X,
     1 * NCALC=*,I5//)
 3020 FORMAT(// 10X, * THIS ITERATION USES ELASTIC STIFFNESS WITH LINER*
     1 , 5X, * NCALC=*, I5//)
      REWIND MTAP1
      REWIND MTAP2
      NUMBLK=0
      NJT=0
      DO 49 N=1,ND
   49 B2(N)=0.0
      IF (NCALC .GT. 3) GO TO 60
      ASSIGN 170 TO NEXT
      DO 50 N=1,ND2
      DO 50 M=1,ND
   50 A(M,N) = 0.0
C     FORM STIFFNESS MATRIX IN BLOCKS
   60 NUMBLK=NUMBLK+1
      NH=NB*(NUMBLK+1)
      NM=NH-NB
      NL=NM-NB+1
      KSHIFT=2*NL-2
C     ADD CONCENTRATED FORCES WITHIN BLOCK
      DO 250 N=NL,NM
      IF(N .GT. NUMNP) GO TO 251
      K=2*N-KSHIFT
      B(K)=UZ(N)+B2(K)
      B2(K)=0.0
```

```
      B(K-1)=UR(N)+B2(K-1)
      B2(K-1)=0.0
  250 CONTINUE
  251 CONTINUE
      IF (LBAD .NE. 0) PRINT 2300
 2300 FORMAT (* PRESSURE B. C. NOT CALCULATED SINCE LBAD .NE. 0*)
      IF (LBAD .NE. 0) GO TO 350
      IF (NIT .GT. 1) GO TO 350
      IF(NUMPC .EQ. 0) GO TO 310
      CALL NPFORC(NUMPC,IJBC,NPBCP,  PSCA ,NL,NM,KSHIFT,NNN,PRATIO,R,
     1  Z,B,NPBC,CODE)
  310 CONTINUE
      IF(NPCAV.EQ.0) GO TO 350
      CALL NPFORC(NCAVPC,IJBCA,NPCAV,PSCAV,NL,NM,KSHIFT,NNN,PRATIO,R,
     1   Z,B,NS,CODE)
  350 CONTINUE
      DO 210 N=1,NUMEL
C
      IF (IX(N,5)) 210,210,65
   65 DO 80 I=1,4
      IF (IX(N,I)-NL) 80,70,70
   70 IF (IX(N,I)-NM) 90,90,80
   80 CONTINUE
      GO TO 210
   90 IF (MJOINT .EQ. 0) GO TO 93
      MTYPE=IX(N,5)
      DO 91 I=1,MJOINT
   91 IF (MTYPE .EQ. MJNT(I)) GO TO 92
      GO TO 93
   92 NJT=NJT+1
      IF (NCALC .GT. 3) GO TO 209
      CALL JTSTIF
      IF (VOL .GT. 0.0) GO TO 165
      LBAD=LBAD+1
      GO TO 209
   93 IF (NCALC .GT. 3) GO TO 95
      IBACK=0
      CALL QUAD(MTAP1)
      IF (VOL .LE. 0.0)  GO TO 209
   95 CONTINUE
   99 CALL LOAD(1,MTAP1)
  144 IF(IX(N,3)-IX(N,4)) 145,165,145
  145 DO 151 II=1,9
      CC=S(II,10)/S(10,10)
      P(II)=P(II)-CC*P(10)
      DO 150 JJ=1,9
  150 S(II,JJ)=S(II,JJ)-CC*S(10,JJ)
  151 CONTINUE
      DO 161 II=1,8
      CC=S(II,9)/S(9,9)
      P(II)=P(II)-CC*P(9)
      IF (NCALC .GT. 3) GO TO 161
      DO 160 JJ=1,8
  160 S(II,JJ)=S(II,JJ)-CC*S(9,JJ)
  161 CONTINUE
  165 DO 166 I=1,4
```

```
  166 LM(I)=2*IX(N,I)-2                                            STIF113C
      DO 200 I=1,4                                                 STIF114C
      DO 200 K=1,2                                                 STIF115C
      II=LM(I)+K-KSHIFT                                            STIF116C
      KK=2*I-2+K                                                   STIF117C
      B(II)=B(II)+P(KK)                                            STIF118C
      GO TO NEXT, (199,170)                                        STIF119C
  170 DO 196 J=1,4                                                 STIF120C
      DO 196 L=1,2                                                 STIF121C
      JJ=LM(J)+L-II+1-KSHIFT                                       STIF122C
      LL=2*J-2+L                                                   STIF123C
      IF(JJ .LE. 0) GO TO 196                                      STIF124C
      A(JJ,II) = A(JJ,II) + S(KK,LL)                               STIF125C
  196 CONTINUE                                                     STIF126C
  199 CONTINUE                                                     STIF127C
  200 CONTINUE                                                     STIF128C
  209 IX(N,5) = -IX(N,5)                                           STIF129C
  210 CONTINUE                                                     STIF130C
      IF (LBAD .NE. 0)  GO TO 405                                  STIF131C
      IF (NCALC .LE. 3) WRITE (MTAP2) A,A2                         STIF132C
      IF (NCALC .GT. 3) READ (MTAP2) A,A2                          STIF133C
C       2. DISPLACEMENT B.C.                                       STIF134C
      DO 400 M=NL,NH                                               STIF135C
      IF (M .GT. NUMNP)  GO TO 401                                 STIF136C
      N=2*M-1-KSHIFT                                               STIF137C
      IF (CODE(M) .LE. 0.0) GO TO 400                              STIF138C
  316 IF (CODE(M) .EQ. 2.0)  GO TO 390                             STIF139C
      CALL MODIFY (A,B,ND2,MBAND,N,UR(M))                          STIF140C
      IF (CODE(M) .EQ. 1.0)  GO TO 400                             STIF141C
  390 N = N + 1                                                    STIF142C
      CALL MODIFY (A,B,ND2,MBAND,N,UZ(M))                          STIF143C
  400 CONTINUE                                                     STIF144C
  401 CONTINUE                                                     STIF145C
C     WRITE BLOCK OF EQUATIONS ON TAPE AND SHIFT UP LOWER BLOCK    STIF146C
      WRITE (9) B                                                  STIF147C
      IF (NCALC .LE. 3) WRITE (9) A                                STIF148C
  405 IF (NM .GE. NUMNP)  GO TO 480                                STIF149C
      IF (LBAD .NE. 0)  GO TO 60                                   STIF150C
      IF (NCALC .GT. 3) GO TO 60                                   STIF151C
      DO 420 N=1,ND                                                STIF152C
      DO 420 M=1,ND                                                STIF153C
      A(M,N)=A2(M,N)                                               STIF154C
  420 A2(M,N)=0.0                                                  STIF155C
      GO TO 60                                                     STIF156C
C                                                                  STIF157C
  480 CONTINUE                                                     STIF158C
      ASSIGN 199 TO NEXT                                           STIF159C
  500 RETURN                                                       STIF160C
 2010 FORMAT(1H1)                                                  STIF161C
      END                                                          STIF162C
```

```
      SUBROUTINE QUAD(MTAP1)
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NTT,
     2    ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1         ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
C
      I=IX(N,1)
      J=IX(N,2)
      K=IX(N,3)
      L=IX(N,4)
      MTYPE=IX(N,5)
      VOL=0.
C
C  FORM STRESS-STRAIN RELATIONSHIP FOR PLANE STRAIN
C
      NEP=0
      CALL STRSTR(STJ1,STJ2,SIGZT,NEP,N,NCALC)
C**********************************************************************
C        FORM QUADRILATERAL STIFFNESS MATRIX
C******************************************************************
      DO 94 M=1,4
      MM=IX(N,M)
   93 RRR(M)=R(MM)
   94 ZZZ(M)=Z(MM)
C
      DO 100 II=1,10
      P(II)=0.0
      DO 95 JJ=1,6
   95 HH(JJ,II)=0.0
      DO 100 JJ=1,10
  100 S(II,JJ)=0.0
      DO 119 II=1,4
      DO 118 IJ=1,31
  118 HSEL(IJ,II)=0.0
      JJ=IX(N,II)
  119 ANGLE(II)=CODE(JJ)/57.3
C
C     FORM BAR STIFFNESS
C
      IF (IX(N,2)-IX(N,3)) 250,240,250
  240 DR=R(J)-R(I)
      DZ=Z(J)-Z(I)
      XL=SQRT(DR**2+DZ**2)
      RRR(5)=(R(I)+R(J))/2.-2.*EE(4)*DZ/XL
      ZZZ(5)=(Z(I)+Z(J))/2.+2.*EE(4)*DR/XL
      IF (NCALC .NE. 1 .OR. NRES .GT. 0) GO TO 242
      IF (NREAD .EQ. 1.OR.NNN.GT.1.OR.NCT.GT.1) GO TO 242
      IF (RO(MTYPE) .EQ. 0. .AND. E(2,MTYPE) .LE. 1.) GO TO 242
      CALL INITST
```

```
  242 CONTINUE
      CALL TRISTF(1,2,5)
      GO TO 130
  250 CONTINUE
      IF (K .NE. L ) GO TO 125
      RRR(5)=(RRR(1)+RRR(2)+RRR(3))/3.0
      ZZZ(5)=(ZZZ(1)+ZZZ(2)+ZZZ(3))/3.0
      IF (NNN .GT. 1 .OR. NCT .GT. 1) GO TO 121
      IF (NCALC .NE. 1 .OR. NRES .GT. 0) GO TO 121
      IF (NREAD .EQ. 1) GO TO 121
      IF (RO(MTYPE) .EQ. 0. .AND. E(2,MTYPE) .LE. 1.) GO TO 121
      CALL INITST
  121 CONTINUE
      CALL TRISTF(1,2,3)
      VOL = XI
      IF (VOL .GT. 0.0)  GO TO 130
C     ERROR RETURN
  122 LBAD = LBAD + 1
      GO TO 135
  125 VOL=0.0
      RRR(5)=(R(I)+R(J)+R(K)+R(L))/4.0
      ZZZ(5)=(Z(I)+Z(J)+Z(K)+Z(L))/4.0
      IF (NRES .GT. 0) GO TO 126
      IF (NREAD .EQ. 1) GO TO 126
      IF (NCALC .NE. 1 .OR. NNN .GT. 1 .OR. NCT .GT. 1) GO TO 126
      IF (RO(MTYPE) .EQ. 0. .AND. E(2,MTYPE) .LE. 1.) GO TO 126
      CALL INITST
  126 CONTINUE
      CALL TRISTF(4,1,5)
      CALL TRISTF(1,2,5)
      CALL TRISTF(2,3,5)
      CALL TRISTF(3,4,5)
      IF (VOL .LE. 0.0)  GO TO 122
  145 DO 140 II=1,6
      DO 140 JJ=1,10
  140 HH(II,JJ)=HH(II,JJ)/4.0
  130 CONTINUE
      WRITE(MTAP1) N,S,HH,RRR(5),ZZZ(5),C,P,HSEL
  135 RETURN
      END
```

```
      SUBROUTINE PRINST (SIG)
      DIMENSION SIG(6)
      CC=(SIG(1)+SIG(2))*0.5
      BB=(SIG(1)-SIG(2))*0.5
      CR=SQRT(BB**2+SIG(3)**2)
      SIG(4)=CC+CR
      SIG(5)=CC-CR
C*****SIG(6) IS AN ANGLE MEASURED FROM R-AXIS TO THE PLANE ON WHICH
C**** MINOR PRINCIPAL STRESS ACTS. POSITIVE IF COUNTERCLOCKWISE
      SIG(6)=45.
      IF (BB .NE. 0.) SIG(6)=28.64789*ATAN2(SIG(3),BB)
      RETURN
      END
```

```
      SUBROUTINE TRISTF(II,JJ,KK)
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2     ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1          ,CRAC(12)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1              ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2              F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      DIMENSION          ZZ(4),DD(3,3),HSAVE(3,10),HS(31)
      EQUIVALENCE (F(1,1),HS(1),HSAVE(1,1))
      IBACK=IBACK+1
C       1. INITIALIZATION
      LM(1)=II
      LM(2)=JJ
      LM(3)=KK
      RR(1)=RRR(II)
      RR(2)=RRR(JJ)
      RR(3)=RRR(KK)
      PR(4)=RRR(II)
      ZZ(1)=ZZZ(II)
      ZZ(2)=ZZZ(JJ)
      ZZ(3)=ZZZ(KK)
      ZZ(4)=ZZZ(II)
   85 DO 100 I=1,6
      DO 90 J=1,10
      F(I,J)=0.0
   90 H(I,J)=0.0
      DO 100 J=1,6
  100 D(I,J)=0.0
C          FORM INTEGRAL(G)T*(C)*(G)
      COMM=RR(2)*(ZZ(3)-ZZ(1))+RR(1)*(ZZ(2)-ZZ(3))+RR(3)*(ZZ(1)-ZZ(2))
      XI = COMM / 2.0
      IF (XI .GT. 0.0)  GO TO 102
      PRINT 1000, II,JJ,KK, N
      LBAD=LBAD+1
 1000 FORMAT (32H ZERO OR NEGATIVE AREA, TRIANGLE3I6,5X,7HELEMENTI5)
      RETURN
  102 VOL=VOL+XI
      D(2,2)=XI   *C(1,1)
      D(2,6) = XI * C(1,2)
      D(3,3)=XI   *C(4,4)
      D(3,5)=XI   *C(4,4)
      D(5,5)=XI   *C(4,4)
      D(6,6)=XI   *C(2,2)
      D(2,3)= XI*C(1,4)
      D(2,5)=D(2,3)
      D(3,6)= XI*C(4,2)
      D(5,6)=D(3,6)
  108 DO 110  I=1,5
      K = I + 1
      DO 110  J = K,6
  110 D(J,I)=D(I,J)
C          FORM  COEFFICIENT-DISPLACEMENT TRANSFORMATION MATRIX
      DD(1,1)=(RR(2)*ZZ(3)-RR(3)*ZZ(2))/COMM
```

```
      DD(1,2)=(RR(3)*ZZ(1)-RR(1)*ZZ(3))/COMM                            TRIS 57C
      DD(1,3)=(RR(1)*ZZ(2)-RR(2)*ZZ(1))/COMM                            TRIS 58C
      DD(2,1)=(ZZ(2)-ZZ(3))/COMM                                        TRIS 59C
      DD(2,2)=(ZZ(3)-ZZ(1))/COMM                                        TRIS 60C
      DD(2,3)=(ZZ(1)-ZZ(2))/COMM                                        TRIS 61C
      DD(3,1)=(RR(3)-RR(2))/COMM                                        TRIS 62C
      DD(3,2)=(RR(1)-RR(3))/COMM                                        TRIS 63C
      DD(3,3)=(RR(2)-RR(1))/COMM                                        TRIS 64C
      DO 120 I=1,3                                                      TRIS 65C
      J=2*LM(I)-1                                                       TRIS 66C
      H(1,J)=DD(1,I)                                                    TRIS 67C
      H(2,J)=DD(2,I)                                                    TRIS 68C
      H(3,J)=DD(3,I)                                                    TRIS 69C
      H(4,J+1)=DD(1,I)                                                  TRIS 70C
      H(5,J+1)=DD(2,I)                                                  TRIS 71C
  120 H(6,J+1)=DD(3,I)                                                  TRIS 72C
C     ROTATE UNKNOWNS IF REQUIRED                                       TRIS 73C
      DO 125 J=1,2                                                      TRIS 74C
      I=LM(J)                                                           TRIS 75C
      IF (ANGLE(I)) 122,125,125                                         TRIS 76C
  122 SINA=SIN(ANGLE(I))                                                TRIS 77C
      COSA=COS(ANGLE(I))                                                TRIS 78C
      IJ=2*I                                                            TRIS 79C
      DO 124 K=1,6                                                      TRIS 80C
      TEM=H(K,IJ-1)                                                     TRIS 81C
      H(K,IJ-1)=TEM*COSA+H(K,IJ)*SINA                                   TRIS 82C
  124 H(K,IJ)= -TEM*SINA+H(K,IJ)*COSA                                   TRIS 83C
  125 CONTINUE                                                          TRIS 84C
C           FORM ELEMENT STIFFNESS MATRIX (H)T*(D)*(H)                  TRIS 85C
      DO 130 J=1,10                                                     TRIS 86C
      DO 130 K=1,6                                                      TRIS 87C
      IF( H(K,J) .EQ. 0.0) GO TO 130                                    TRIS 88C
  128 DO 129 I=1,6                                                      TRIS 89C
  129 F(I,J)=F(I,J)+D(I,K)*H(K,J)                                       TRIS 90C
  130 CONTINUE                                                          TRIS 91C
      DO 140 I=1,10                                                     TRIS 92C
      DO 140 K=1,6                                                      TRIS 93C
      IF (H(K,I) .EQ. 0.0) GO TO 140                                    TRIS 94C
  138 DO 139 J=1,10                                                     TRIS 95C
  139 S(I,J)=S(I,J)+H(K,I)*F(K,J)                                       TRIS 96C
  140 CONTINUE                                                          TRIS 97C
C        FORM RESIDUAL LOAD MATRIX                                      TRIS 98C
C**** FORM RESIDUAL LOAD MATRIX                                         TRIS 99C
  150 DO 160 I=1,10                                                     TRIS100C
      HSAVE(1,I)=H(2,I)                                                 TRIS101C
      HSAVE(2,I)=H(6,I)                                                 TRIS102C
  160 HSAVE(3,I)=H(3,I)+H(5,I)                                          TRIS103C
C     ACCELERATION LOADS                                                TRIS104C
      IF (NCALC .NE. 1.OR.NCT.GT.1) GO TO 171                           TRIS105C
      IF(NREAD.EQ.1.OR.NNN.GT.1) GO TO 171                              TRIS106C
      IF (ACELX.EQ. 0.0 .AND. ACELY.EQ. 0.0) GO TO 171                  TRIS107C
      COMM = RO(MTYPE) * XI / 3.0                                       TRIS108C
      DO 170 I=1,3                                                      TRIS109C
      J=2*LM(I)-1                                                       TRIS110C
      P(J) = P(J)-ACELX*COMM                                            TRIS111C
  170 P(J+1) = P(J+1) - ACELY*COMM                                      TRIS112C
```

```
  171 CONTINUE
C     FORM STRAIN TRANSFORMATION MATRIX
  400 DO 410 I=1,6
      DO 410 J=1,10
  410 HH(I,J)=HH(I,J)+H(I,J)
      HS(31)=XI
      DO 420 I=1,31
  420 HSEL(I,IBACK)=HS(I)
      RETURN
      END
```

```
      SUBROUTINE MODIFY(A,B,NEQ,MBAND,N,U)
      DIMENSION A(90,180),B(180)
C
C
      DO 250 M=2,MBAND
      K=N-M+1
      IF (K .LE. 0)  GO TO 235
      IF (A(M,K) .EQ. 0.0)  GO TO 235
      B(K) = B(K) - A(M,K) * U
      A(M,K) = 0.0
  235 K=N+M-1
      IF (K .GT. NEQ) GO TO 250
      IF (A(M,N) .EQ. 0.0)  GO TO 250
      B(K) = B(K) - A(M,N) * U
      A(M,N) = 0.0
  250 CONTINUE
      A(1,N) = 1.0
      B(N)=U
C
      RETURN
      END
```

```
      SUBROUTINE JTSTIF
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2     ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1         ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1              ANGLE(4),XI,HH(6,10),C(4,4),FE(4),H(6,10),D(6,6),
     2              F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      COMMON /JNT/ FN(450),FT(450),NJT
      DIMENSION ESTIF(10,10),PPP(8),TR(2,2),Y(4,4)
      EQUIVALENCE (L,VOL),(S(1,1),ESTIF(1,1))
      REAL KS,KN,L
      DATA Y/2.,1.,-1.,-2.,1.,2.,-2.,-1.,-1.,-2.,2.,1.,-2.,-1.,1.,2./
      II=IX(N,1)
      JJ= IX(N,2)
      DR=R(JJ)-R(II)
      DZ=Z(JJ)-Z(II)
      L=SQRT(DR*DR+DZ*DZ)
      IF(L.EQ.0.) GO TO 201
      MTYPE=IX(N,5)
      IF( NRES .GT. 0) GO TO 40
      IF(NCALC.NE.1) GO TO 40
      IF(NNN.GT.1.OR.NCT.GT.1) GO TO 40
      IF(NREAD .EQ. 1) GO TO 40
      RRR(5)=0.5*(R(JJ)+R(II))
      ZZZ(5)=0.5*(Z(JJ)+Z(II))
      CALL INITST
   40 CONTINUE
C** MATERIAL PROPERTIES
      KN=E(1,MTYPE)
      KS=E(2,MTYPE)
   50 COMS=KS*L/6.
      COMN=KN*L/6.
C         INITIALIZE
      DO 100 II=1,8
      P(II)=0.0
      PPP(II)=0.0
      DO 100 JJ=1,8
  100 ESTIF(II,JJ)=0.0
C       DEVELOP RESIDUAL STRESS CONTRIBUTIONS TO THE LOAD VECTOR
C THE FOLLOWING SIGN CONVENTION IS ADOPTED.THE NORMAL STRESS IS POSITIVE
C DIRECTED OUTWARDS THE ELEMENT ON THE FACE (II,JJ).THE SHEAR STRESS IS
C WHEN DIRECTED FROM JJ TO II AND LL TO KK   ON   THE ELEMENT.
      TR(1,1)=DR/L
      TR(1,2)=DZ/L
      IF(NCALC.NE.1) GO TO 162
      IF(NNN.GT.1.OR.NCT.GT.1) GO TO 162
      IF (NRES .EQ. 0 .OR. NRES .EQ. 2) GO TO 162
      SC = TR(1,1) * TR(1,2)
      S2 = TR(1,2)  ** 2
      C2 = TR(1,1) ** 2
  111 RSTRS(1)=STRS(N,1)*S2+STRS(N,2)*C2-2.*STRS(N,3)*SC
```

```
      RSTRS(2)=(STRS(N,2)-STRS(N,1))*SC-(STRS(N,3)*(S2-C2))
      EL2 = L / 2.0
      DO 160 I=1,4
      J=2*I-1
      PPP(J) = RSTRS(1) * EL2
      J=2*I
  160 PPP(J) = RSTRS(2) * EL2
      DO 161 I=1,4
  161 PPP(I)=-PPP(I)
  162 CONTINUE
      DO 200 II=1,4
      IS=2*II-1
      IN=2*II
      DO 200 JJ=1,4
      JS = 2*JJ-1
      JN = 2*JJ
      ESTIF(IS,JS)=COMS*Y(II,JJ)
  200 ESTIF(IN,JN)=COMN*Y(II,JJ)
C     ROTATE TO GLOBAL COORDINATES
      TR(2,1) = -TR(1,2)
      TR(2,2) = TR(1,1)
      IF(TR(1,1).EQ.1.) GO TO 405
      DO 400 NN=1,4
      DO 410 II=1,8
      JJ= 2*NN-1
      TEMP = ESTIF(II,JJ)
      DO 410 KK=1,2
      ESTIF(II,JJ)=TEMP*TR(1,KK)+ESTIF(II,2*NN)*TR(2,KK)
  410 JJ=JJ+1
      DO 420 II=1,8
      JJ=2*NN-1
      TEMP =ESTIF(JJ,II)
      DO 420 KK=1,2
      ESTIF(JJ,II)=TR(1,KK)*TEMP+TR(2,KK)*ESTIF(2*NN,II)
  420 JJ=JJ+1
  400 CONTINUE
  405 CONTINUE
      IF (NRES .EQ. 0 .OR. NRES .EQ. 2) GO TO 402
      DO 401 I=1,4
      J=2*I-1
      II=2*I
      P(J)=-PPP(J)*TR(1,2)+PPP(II)*TR(2,2)
  401 P(II)= PPP(J)*TR(2,2)+PPP(II)*TR(1,2)
  402 CONTINUE
      RETURN
  201 PRINT 2090, N
      RETURN
 2090 FORMAT(14H BAD JOINT,N= I3/)
      END
```

```
      SUBROUTINE BANSOL (NNN,NIT,NCALC)
      COMMON /BANARG/ B(90),B2(90),A(90,90),A2(90,90),MBAND,ND2,NUMBLK,
     1                MBMAX,NBB ,MTAP1,MTAP2
      EQUIVALENCE (MM,MBAND)
      NN=MBMAX
      NTAPE=MTAP2
      IF (NCALC .LE. 3) NTAPE=9
      REWIND 91
   99 REWIND 9
      NB=0
      GO TO 150
C     REDUCE EQUATIONS BY BLOCKS
C     1. SHIFT BLOCK OF EQUATIONS
  100 NB=NB+1
      DO 125 N=1,NN
      B(N) = B2(N)
      B2(N) = 0.0
      DO 125 M=1,MM
      A(M,N) = A2(M,N)
  125 A2(M,N) = 0.0
C     2. READ NEXT BLOCK OF EQUATIONS INTO CORE
      IF (NUMBLK-NB) 150,200,150
  150 READ (9) B2
      READ (NTAPE) A2
      IF (NB .EQ. 0) GO TO 100
C     3. REDUCE BLOCK OF EQUATIONS
  200 IF (NCALC .GT. 3) GO TO 2000
      DO 300 N=1,NN
      IF (A(1,N) .EQ. 0.0)  GO TO 300
      DO 275 L=2,MM
      IF (A(L,N) .EQ. 0.0)  GO TO 275
      C = A(L,N) / A(1,N)
      I=N+L-1
      J=0
      DO 250 K=L,MM
      J=J+1
  250 A(J,I) = A(J,I) - C * A(K,N)
      A(L,N) = C
  275 CONTINUE
  300 CONTINUE
      WRITE(MTAP2) A
 2000 DO 2300 N=1,NN
      IF (A(1,N) .EQ. 0.0) GO TO 2300
      DO 2275 L=2,MBAND
      IF (A(L,N) .EQ. 0.0) GO TO 2275
      I=N+L-1
      B(I)=B(I)-A(L,N)*B(N)
 2275 CONTINUE
      B(N)=B(N)/A(1,N)
 2300 CONTINUE
C**** WRITE BLOCK OF REDUCED EQUATIONS ON TAPE 91
      IF(NUMBLK .EQ. NB) GO TO 399
  375 WRITE (91) B
      GO TO 100
C     BACK-SUBSTITUTION
  399 BACKSPACE MTAP2
```

```
      NTAPE=MTAP2
  400 DO 450 M=1,NN
      N=NN+1-M
      DO 425 K=2,MM
      L=N+K-1
  425 B(N) = B(N) - A(K,N) * B(L)
      B2(N) = B(N)
  450 A2(NB,N) = B(N)
      NB=NB-1
      IF (NB .EQ. 0) GO TO 500
  475 BACKSPACE MTAP2
      BACKSPACE 91
      READ(MTAP2) A
      READ (91) B
      BACKSPACE MTAP2
      BACKSPACE 91
      GO TO 400
C     ORDER UNKNOWNS IN B ARRAY
  500 K=0
      DO 600 NB=1,NUMBLK
      DO 600 N=1,NN
      K=K+1
  600 B(K) = A2(NB,N)
      RETURN
      END
```

```
      SUBROUTINE STRESS
      COMMON /    / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2    ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1       ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /BANARG/ B(180),A(90,180),MBAND,ND2,NUMBLK,MBMAX,NB
     1,MTAP1,MTAP2
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      COMMON /JNT/ FN(450),FT(450),NJT
      DIMENSION  TP(6), FY(900)
      DIMENSION STRN(900,3)
      EQUIVALENCE (STRN,A(6500))
      EQUIVALENCE (FY,A(4000))
C     COMPUTE ELEMENT STRESSES
      REWIND MTAP1
      TENMAX=0.0
      NTEN=0
      MPRINT =0
      NEXIT=0
      DO 100 N=1,NUMEL
  100 IX(N,5)=IABS(IX(N,5))
      NELMJ=NUMEL-NJT
      DO 300 MN=1,NELMJ
      CALL LOAD(9,MTAP1)
      DO 120 I=1,4
      II=2*I
      JJ=2*IX(N,I)
      P(II-1)=B(JJ-1)
  120 P(II)=B(JJ)
      DO 150 I=1,2
      RR(I)=P(I+8)
      DO 150 K=1,8
  150 RR(I)=RR(I)-S(I+8,K)*P(K)
      COMM=S(9,9)*S(10,10)-S(9,10)*S(10,9)
      IF (COMM) 155,160,155
  155 P(9)=(S(10,10)*RR(1)-S(9,10)*RR(2))/COMM
      P(10)=(-S(10,9)*RR(1)+S(9,9)*RR(2))/COMM
  160 DO 170 I=1,6
      TP(I)=0.0
      DO 170 K=1,10
  170 TP(I)=TP(I)+HH(I,K)*P(K)
      RR(1)=TP(2)
      RR(2)=TP(6)
      RR(3)=0.0
      RR(4)=TP(3)+TP(5)
      IF (NCALC .NE. 1 .OR. NREAD .EQ. 1) GO TO 175
      IF(NNN.GT.1.OR.NCT.GT.1) GO TO 175
      IF(NRES.GT.0) GO TO 173
      STRS(N,4)=STRS(N,1)
  173 CONTINUE
      IF(ABS(STRS(N,1)).LT.0.001.AND.ABS(STRS(N,2)).LT.0.001.AND.
```

```
     1      ABS(STRS(N,3)) .LT.0.001) GO TO 175
      STJ1=STRS(N,1)+STRS(N,2)+STRS(N,4)
      STJ2=((STRS(N,1)-STRS(N,2))**2+(STRS(N,2)-STRS(N,4))**2+
     1(STRS(N,4)-STRS(N,1))**2)/6.+STRS(N,3)**2
      FY(N)=E(7,MTYPE)*STJ1+SQRT(STJ2)-E(8,MTYPE)
  175 DO 180 I=1,4
      DSIG(I)=0.0
      DO 180 K=1,4
  180 DSIG(I)=DSIG(I)+C(I,K)*RR(K)
      DSIGZ=DSIG(3)
      DSIG(3)=DSIG(4)
      DO 181 I=1,3
  181 SIG(I)=RSTRS(I)+DSIG(I)
      DO 400 J=1,3
      STRS(N,J)=STRS(N,J)+SIG(J)
      SIG(J)=STRS(N,J)
  400 CONTINUE
      STRN(N,1)=RR(1)+STRN(N,1)
      STRN(N,2)=RR(2)+STRN(N,2)
      STRN(N,3)=RR(4)+STRN(N,3)
      DO 450 I=1,2
      TP(I)=STRN(N,I)
  450 CONTINUE
      TP(3)=0.5*STRN(N,3)
      CALL PRINST(TP)
      TP(3)=STRN(N,3)
C     OUTPUT STRESSES
C     CALCULATE PRINCIPAL STRESSES
      CALL PRINST(SIG)
      IF(MTAG(N).EQ.0) GO TO 263
      IF(E(2,MTYPE) .LT. 2.)GO TO 263
      IF (MTENS .EQ. 0) GO TO 570
      DO 560 J=1,MTENS
  560 IF (MTYPE .EQ. MNTEN(J)) GO TO 263
  570 CONTINUE
      IF (SIG(4) .LE. TENMAX) GO TO 200
      TENMAX=SIG(4)
      NTEN=N
  200 CONTINUE
      TMAX=E(1,MTYPE)
      IF (MTAG(N) .GE. 2) GO TO 250
      IF (SIG(4) .LE. TMAX) GO TO 263
      MTAG(N)=2
  250 IF (MTAG(N) .EQ. 3) GO TO 263
      IF (SIG(5) .LE. TMAX) GO TO 263
      MTAG(N)=3
  263 CONTINUE
C*******
      IF (MPRINT .NE. 0) GO TO 110
  105 PRINT 2000
      PRINT 2400
      PRINT 2300
      MPRINT=25
  110 MPRINT=MPRINT-1
      MPRINT=MPRINT+1
      PRINT 2001,N,RRR(5),ZZZ(5),(SIG(I),I=1,6),TP
```

```
 117 CALL EPLAST(NEXIT)                                                   STRE113C
     PRINT 2100,MTAG(N),(SIG(I),I=1,3),STRS(N,4),(SEP(N,I),I=1,3),FY(N)STRE114C
 300 CONTINUE                                                             STRE115C
     IF (NIT .NE. ITN(NNN)) GO TO 350                                     STRE116C
     PRINT 2002,TENMAX,NTEN                                               STRE117C
 350 CONTINUE                                                             STRE118C
     IF (NEXIT .GT. 0) CALL EXIT                                          STRE119C
     RETURN                                                               STRE120C
2000 FORMAT (7H1EL.NO.   7X  1HX 7X 1HY 4X 8HX-STRESS 9X 8HY-STRESS 7X   STRE121C
    19HXY-STRESS 5X 10HMAX-STRESS 5X 10HMIN-STRESS ,4X,5HANGLE           STRE122C
    2          /15X,8HX-STRAIN,9X,8HY-STRAIN,7X,9HXY-STRAIN,5X,10HMAX-STRSTRE123C
    3AIN,4X,10HMIN-STRAIN,3X,5HANGLE )                                   STRE124C
2001 FORMAT (I7,2F8.2,1P5E15.4,0P1F7.2   /10X,1P5E15.4,0P1F7.2)          STRE125C
2002 FORMAT (1H110X,17HMAXIMUM TENSION =1PE10.3,11H AT ELEMENTI5/)       STRE126C
2003 FORMAT (I6,1P9E14.4)                                                 STRE127C
2100 FORMAT (5X,1H(,I2,1H),4X,1P3E15.4,1P5E13.4)                          STRE128C
2200 FORMAT(10X,8HR-STRESS,6X,8HT-STRESS,3X,9HRT-STRESS,5X,8HR-STRAIN,  STRE129C
    1 6X,8HT-STRAIN,6X,9HRT-STRAIN)                                       STRE130C
2300 FORMAT(4X,4HMTAG,2X,7HCURRENT,2X,8HX-STRESS,4X,8HY-STRESS,7X,9HXY-STRE131C
    1STRESS,8X,5HSIGZZ,3X,15HEXCESS X-STRESS,2X,8HY-STRESS,3X,9HXY-STRESTRE132C
    2SS,3X,14HYIELD FUNCTION)                                             STRE133C
2400 FORMAT(4X,*YIELD FUNCTION(FY)*,*  PREVIOUS*,4X,*PRESENT*,3X,*STRNTSTRE134C
    1H RATIO*,* EXPECTED FY*,* OVER RELAXATION*,* SIGZZ*)                 STRE135C
     END                                                                  STRE136C
```

```
      SUBROUTINE INITST                                                  INIT   1C
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,   INIT   2C
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,ICALC,IBACK,MJOINT,MTENS,NIT,        INIT   3C
     2    ITN(20),PRATIO(20),NISTOP,NHEAD,NSTSRT,NANALY                   INIT   4C
     3,NCT,NCONST,NPBCP,NCAVPC                                            INIT   5C
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)      INIT   6C
     1        ,CRAC(12)                                                   INIT   7C
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)INIT   8C
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),     INIT   9C
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),      INIT  10C
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ          INIT  11C
      STRS(N,2)=REFPRS+RO(MTYPE)*(ZZZ(5)-DEPTH)                          INIT  12C
      IF (NRES .EQ. -1) STRS(N,2)=0.0                                     INIT  13C
      STRS(N,1)=AKO(MTYPE)*STRS(N,2)                                      INIT  14C
      RETURN                                                              INIT  15C
      END                                                                 INIT  16C
```

```
      SUBROUTINE LOAD(JUMP,MTAP1)
C**** FROM STIFF JUMP=1
C**** FROM STRESS JUMP=9 - ONLY NEED P(9) AND P(10)
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2    ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1        ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1              ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2              F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      DIMENSION HS(31),X(3,10)
      EQUIVALENCE (HS(1),X(1,1))
      IF (NCALC .LE. 3 .AND. JUMP .EQ. 1) GO TO 500
      READ(MTAP1) NI,S,HH,RRR(5),ZZZ(5),C,P,HSEL
      N=NI
  500 CONTINUE
      MTYPE=IX(N,5)
      IF (NCALC .EQ. 1 .AND. NCT .EQ. 1 .AND. NNN .EQ. 1
     1.AND. NIT .EQ. 1) GO TO 105
      IF (NCALC .EQ. 1) GO TO 105
C**** DO NOT CLEAR GRAVITY FOR FULL CALCULATION
      DO 100 I=1,10
  100 P(I)=0.0
  105 DO 110 I=1,4
  110 RSTRS(I)=0.0
      IF(MTAG(N).EQ.0) GO TO 400
      IF(E(2,MTYPE) .LT. 2.) GO TO 400
      IF (NCALC .NE. 1.OR.NNN.GT.1) GO TO 200
      IF (NREAD .EQ. 1.OR.NCT.GT.1) GO TO 200
      IF (NRES .EQ. 0 .OR. NRES .EQ. 2) GO TO 200
      DO 120 I=1,3
  120 RSTRS(I)=-STRS(N,I)
      RSTRS(4)=RSTRS(3)
      IF (JUMP .EQ. 1) GO TO 200
      DO 270 I=1,3
      STRS(N,I)=STRS(N,I)+RSTRS(I)
  270 CONTINUE
  200 CONTINUE
      DO 310 I=1,3
  310 RSTRS(I)=RSTRS(I)-SEP(N,I)
C
      IF (IX(N,2) .EQ. IX(N,3)) GO TO 340
      IF (IX(N,3) .EQ. IX(N,4)) GO TO 340
      II=4
      GO TO 350
  340 II=1
  350 DO 360 J=1,II
      DO 355 IJ=1,31
  355 HS(IJ)=HSEL(IJ,J)
      DO 360 I=JUMP,10
  360 P(I)= P(I)-HS(31)*(RSTRS(1)*X(1,I)+RSTRS(2)*X(2,I)
     1                   + RSTRS(3)*X(3,I))
  400 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE JTSTR
      COMMON /   / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2     ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPBCP,NCAVPC
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /BANARG/ B(180),A(90,180),MBAND,ND2,NUMBLK,MBMAX,NB
     1,MTAP1,MTAP2
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1          ,CRAC(12)
      COMMON /NPDATA/ R(999),Z(999),CODE(999),UR(999),UZ(999)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1              ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2              F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      COMMON /JNT/ FN(450),FT(450),NJT
      DIMENSION DISP(900,2),V(4),U(4)
      EQUIVALENCE (DISP,A(2000))
      REAL L,KN,KS
      PRINT 1001
C  ESTABLISH DISPLACEMENT ALONG AND NORMAL TO JOINT
      IF (NRES .EQ. -1) PUNCH 2000,HED,NJT
 2000 FORMAT (8A10,/*  INITIAL STRESSES FOR JOINTS*/I5)
      NJT=0
      DO 500 N=1,NUMEL
      MAT = IX(N,5)
      IF (MJOINT .EQ. 0) GO TO 500
      DO 50 I=1,MJOINT
   50 IF (MAT .EQ. MJNT(I)) GO TO 60
      GO TO 500
   60 CONTINUE
      KN=E(1,MAT)
      KS=E(2,MAT)
      NJT=NJT+1
      IF(MTAG(N).GT.0) GO TO 70
      FN(NJT)=0.
      FT(NJT)=0.
      GO TO 500
   70 CONTINUE
      I=IX(N,1)
      J= IX(N,2)
      DR=R(J)-R(I)
      DZ=Z(J)-Z(I)
      RRJ=0.5*(R(J)+R(I))
      ZZJ=0.5*(Z(J)+Z(I))
      L=SQRT(DR*DR+DZ*DZ)
      DR=DR/L
      DZ=DZ/L
      DO 100 II=1,4
      K=IX(N,II)
      V(II)=-B(2*K-1)*DZ+B(2*K)*DR
  100 U(II)=B(2*K-1)*DR+B(2*K)*DZ
C   COMPUTE EFFECTIVE STRAIN
C EPSN POSITIVE MEANS JOINT IS OPEN
C EPST POSITIVE MEANS (KK,LL) MOVES ALONG U+ MORE THAN (II,JJ)
  200 EPST=0.5*(U(4)-U(1)+U(3)-U(2))
      EPSN=0.5*(V(4)-V(1)+V(3)-V(2))
```

```
C  COMPUTE NORMAL AND SHEAR FORCE PER UNIT LENGTH AND CALCULATE STRENGTH
C INITIAL STRESSES INPUT ARE ALWAYS COMPRESSIVE (NEGATIVE)
      FNRM=0.0
      FTRM=0.0
      EPNRM=0.0
      IF(NCALC.NE.1.OR.NNN.GT.1) GO TO 300
      IF (NRES .EQ. 1 .OR. NRES .EQ. 2) GO TO 300
      IF (NREAD .EQ. 1.OR.NCT.GT.1) GO TO 300
      C2=DR**2
      S2=DZ**2
      SC=DR*DZ
      FNRES=STRS(N,1)*S2+STRS(N,2)*C2-STRS(N,3)*2.*SC
      FTRES=(-STRS(N,1)+STRS(N,2))*SC-STRS(N,3)*(S2-C2)
      FN(NJT)=FNRES
      FT(NJT)=FTRES
  300 DO 310 II=1,4
      K=IX(N,II)
      V(II)=-DISP(K,1)*DZ+DISP(K,2)*DR
  310 U(II)=DISP(K,1)*DR+DISP(K,2)*DZ
      TEPST=0.5*(U(4)-U(1)+U(3)-U(2))
      TEPSN=0.5*(V(4)-V(1)+V(3)-V(2))
      FN(NJT)=KN*EPSN+FN(NJT)
      IF (FN(NJT) .LE. 0.) GO TO 201
      FNRM=FN(NJT)
      GO TO 202
  201 IF (TEPSN .GE. 0.) GO TO 202
      IF (TEPSN .GT. E(5,MAT)) GO TO 202
C
C E(5,MAT) SHOULD BE INPUT AS A NEGATIVE QUANTITY
C
      FNRM=KN*(TEPSN-E(5,MAT))
      EPNRM=TEPSN-E(5,MAT)
  202 FT(NJT)=KS*EPST+FT(NJT)
      STREN = 0.
      IF (FN(NJT) .GE. 0.) GO TO 205
      IF (TEPST .EQ. 0.) GO TO 210
      STREN = E(3,MAT)+ABS(FN(NJT))*TAN(E(4,MAT)*0.01745329)
      IF (ABS(FT(NJT)) .LT. STREN ) GO TO 210
      IF ( FT(NJT) .LT. 0.) GO TO 203
      FTRM=FT(NJT)-STREN
      GO TO 210
  203 FTRM= FT(NJT)+STREN
      GO TO 210
  205 FTRM=FT(NJT)
  210 CONTINUE
  220 PRINT 1000,N,RRJ,ZZJ,FN(NJT),FT(NJT),TEPSN,TEPST,EPSN,EPST,FNRM,
     1               FTRM
      IF (NRES .EQ. -1) PUNCH 2100 ,FN(NJT),FT(NJT),N
 2100 FORMAT (2F20.5,I5)
      IF (EPNRM .EQ. 0.) GO TO 421
      EPNRM=EPNRM*0.5
      DO 420 II=1,4
      K=IX(N,II)
      SIGNT=1.
      IF (II .GT. 2) SIGNT=-1.
      IF (CODE(K) .EQ. 3) GO TO 420
```

```
      IF(CODE(K) .EQ. 1) GO TO 415                                      JTST113C
      DISP(K,1)=DISP(K,1)-SIGNT*EPNRM*DZ                                JTST114C
      IF (CODE(K) .EQ. 2) GO TO 420                                     JTST115C
  415 DISP(K,2)=DISP(K,2)+SIGNT*EPNRM*DR                                JTST116C
  420 CONTINUE                                                          JTST117C
  421 CONTINUE                                                          JTST118C
      IF (FNRM .EQ. 0. .AND. FTRM .EQ. 0. ) GO TO 431                   JTST119C
      IF(EPNRM .NE. 0.) GO TO 425                                       JTST120C
      FN(NJT) = FN(NJT) - FNRM                                          JTST121C
  425 FT(NJT) = FT(NJT) - FTRM                                          JTST122C
      FNRM=FNRM*0.5                                                     JTST123C
      FTRM=FTRM*0.5                                                     JTST124C
      DZ=DZ*L                                                           JTST125C
      DR=DR*L                                                           JTST126C
      DO 430 II=1,4                                                     JTST127C
      K=IX(N,II)                                                        JTST128C
      SIGNT=1.                                                          JTST129C
      IF (II .GT. 2) SIGNT=-1.                                          JTST130C
      IF (CODE(K) .EQ. 3) GO TO 430                                     JTST131C
      IF (CODE(K) .EQ. 1) GO TO 428                                     JTST132C
      UR(K)=UR(K)+SIGNT*(FNRM*DZ-FTRM*DR)                               JTST133C
      IF (CODE(K) .EQ. 2) GO TO 430                                     JTST134C
  428 UZ(K)=UZ(K)-SIGNT*(FNRM*DR+FTRM*DZ)                               JTST135C
  430 CONTINUE                                                          JTST136C
  431 CONTINUE                                                          JTST137C
  500 CONTINUE                                                          JTST138C
      RETURN                                                            JTST139C
 1000 FORMAT (I5,2F8.2,8E13.5)                                          JTST140C
 1001 FORMAT(1H1,6HEL NO.,4X,1HX,5X,1HY,3X,12HNORMAL STRS.,13H TANGTL STJTST141C
     1RS.,14H TOT.NOR.DISP.,14H TOT.TANGDISP.,14H DEL.NOR.DISP.,14H DEL.JTST142C
     2TANGDISP.,12H REMOVD SIGN,12H REMOVD SIGT //)                     JTST143C
      END                                                               JTST144C
```

```
      SUBROUTINE EPLAST (NEXIT)
C**** THIS SUBROUTINE CALCULATES STRESS IN ELEMENT AT YIELD
      COMMON /    / NUMNP,NUMEL,NUMMAT,NUMPC,ACELX,ACELY,HED(8),NNN,NP,
     1 NPCAV,REFPRS,DEPTH,NRES,N,VOL,NCALC,IBACK,MJOINT,MTENS,NIT,
     2     ITN(20),PRATIO(20),NISTOP,NREAD,NSTSRT,NANALY
     3,NCT,NCONST,NPSCP,NCAVPC
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1        ,CRAC(12)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /BANARG/ B(180),A(90,180),MBAND,ND2,NUMBLK,MBMAX,NB
     1,MTAP1,MTAP2
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1               ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2               F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      DIMENSION FY(900), SIGP(5)
      EQUIVALENCE (FY,A(4000))
      RF=0.
      OVRX=1.
      IF(E(2,MTYPE) .LT. 2.) GO TO 700
      DO 100 I=1,3
      RSTRS(I)=0.0
  100 SEP(N,I)=0.0
      IF(MTAG(N).EQ.0) GO TO 700
      DO 101 I=1,5
  101 SIGP(I)=SIG(I)
      TENS1=0.0
      TENS2=0.0
      PN=E(3,MTYPE)
      IF (MTENS .EQ. 0) GO TO 110
      DO 105 I=1,MTENS
  105 IF (MTYPE .EQ. MNTEN(I)) GO TO 150
C**** CALCULATE TENSILE STRESS IN NO TENSION ELEMENTS
  110 CONTINUE
      EPSM=SIG(6)/57.29577
      S2=SIN(EPSM)**2
      C2=COS(EPSM)**2
      CS=SIN(EPSM)*COS(EPSM)
      TMAX=E(1,MTYPE)
      IF (MTAG(N) .EQ. 1) GO TO 150
      IF (SIG(4) .GT. CRAC(MTYPE)*TMAX .AND. MTAG(N) .NE. 4) GO TO 112
      IF (SIG(4) .LE. TMAX) GO TO 150
  112 TENS1=SIG(4)
      IF (MTAG(N) .EQ. 3 .AND. SIG(5) .GT. 0.) GO TO 114
      IF (SIG(5) .LE. TMAX) GO TO 120
  114 TENS2=SIG(5)
  120 CONTINUE
      T1=TENS1
      PN2=1.+PN*PN
      TENS1=TENS1*PN2+TENS2*PN*PN2
      TENS2=TENS2*PN2+T1*PN*PN2
      RSTRS(1)=TENS1*C2+TENS2*S2
      RSTRS(2)=TENS1*S2+TENS2*C2
      RSTRS(3)=(TENS1-TENS2)*CS
      DO 125 I=1,3
      IF (ABS(RSTRS(I)) .GT. 1) NISTOP=NISTOP+1
  125 SIG(I)=SIG(I)-RSTRS(I)
```

```
      CALL PRINST(SIG)
  150 IF (ABS(SIG(1)) .LE. 1.E-10 .AND. ABS(SIG(2)) .LE. 1.E-10)GO TO600
      ANG=E(6,MTYPE)
      ALP=E(7,MTYPE)
      FK=E(8,MTYPE)
      CALL PRINST(DSIG)
      IF (NANALY .EQ. 0 .AND. FY(N) .LT. 0.) GO TO 160
      IF(NSPT .EQ. 1 .AND. NIT .GT. 2) GO TO 160
      IF(NANALY .GE. 1 .AND. NIT .GT. 1 .AND. NSPT .EQ. 0) GO TO 160
      IF (FY(N) .LT. 0.) GO TO 160
      DSIGZ=0.5*(DSIG(4)+DSIG(5)-(DSIG(4)-DSIG(5))*SIN(ANG))
  160 SIGZT=STRS(N,4)+DSIGZ-PN*(RSTRS(1)+RSTRS(2))
      SGZTT=SIGZT
      SGZI=SIGZT
      STJ1=SIG(4)+SIG(5)+SIGZT
      STJ2=((SIG(4)-SIG(5))**2+(SIG(5)-SIGZT)**2+(SIGZT-SIG(4))**2)/6.
      FAK=ALP*STJ1-FK
      SRJ2=SQRT(STJ2)
      FY2=FAK+SRJ2
C
      IF (FY2 .GE. 0.0) GO TO 165
      IF(MTAG(N).EQ.6) MTAG(N)=3
      IF(MTAG(N).EQ.5) MTAG(N)=2
      IF(MTAG(N).EQ.4) MTAG(N)=1
      STRS(N,4)=STRS(N,4)+PN*(DSIG(4)+DSIG(5))-PN*(RSTRS(1)+RSTRS(2))
      GO TO 570
  165 CONTINUE
      IF(MTAG(N).EQ.1) MTAG(N)=4
      IF(MTAG(N).EQ.2) MTAG(N)=5
      IF(MTAG(N).EQ.3) MTAG(N)=6
      IF (ABS(FAK) .LT. 0.0001) GO TO 170
      FY2RT=-FY2/FAK
      GO TO 175
  170 FY2RT=10.
  175 CONTINUE
      IF (FY2RT .GT. 0.05) NISTOP=NISTOP+1
      IF (FY2RT .GE. 0.15) CFY=1.0
      IF (FY2RT .LT. 0.15 .AND. FY2RT .GE. 0.1) CFY=0.75
      IF (FY2RT .LT. 0.10) CFY=0.50
      IF (FY(N) .GE. 0.) GO TO 300
C**** F(SIGO) LT ZERO BUT F(SIG) GE ZERO
C
      DJ1=DSIG(4)+DSIG(5)+DSIGZ
      DJ2=((DSIG(4)-DSIG(5))**2+(DSIG(5)-DSIGZ)**2+
     1      (DSIGZ-DSIG(4))**2)/6.
      DF=ALP*DJ1+SQRT(DJ2)
      IF (ABS(DF) .LE. 1.0E-10) GO TO 500
      RF=FY2/DF
      DO 250 I=1,3
  250 SIG(I)=SIG(I)-DSIG(I)*RF
      SGZTT=         (1.-RF)*PN*(DSIG(4)+DSIG(5))-PN*(RSTRS(1)+RSTRS(2))
     1+STRS(N,4)
      SIGZT=SGZTT+RF*0.5*(DSIG(4)+DSIG(5)-(DSIG(4)-DSIG(5))*SIN(ANG))
      STJ1=SIG(1)+SIG(2)+SGZTT
      STJ2=((SIG(1)-SIG(2))**2+(SIG(2)-SGZTT)**2+(SGZTT-SIG(1))**2)/6.+
     1       SIG(3)**2
```

```
  300 CONTINUE                                                          EPLA113C
      NEP=1                                                             EPLA114C
      NST=0                                                             EPLA115C
  310 CALL STRSTR(STJ1,STJ2,SGZTT,NEP,N,NCALC,NEXIT)                    EPLA116C
      DO 350 I=1,4                                                      EPLA117C
      SIG(I)=0.0                                                        EPLA118C
      DO 350 K=1,4                                                      EPLA119C
  350 SIG(I)=SIG(I)+C(I,K)*RR(K)                                        EPLA120C
      DSIGZZ=SIG(3)                                                     EPLA121C
      SIG(3)=SIG(4)                                                     EPLA122C
  360 CONTINUE                                                          EPLA123C
C                                                                       EPLA124C
C**** CALCULATE EXCESS STRESS TO BE SUPPORTED BY BODY FORCES            EPLA125C
C                                                                       EPLA126C
      DO 370 I=1,3                                                      EPLA127C
  370 SEP(N,I)=DSIG(I)-SIG(I)                                           EPLA128C
      DSIGZ3=DSIGZ-DSIGZZ                                               EPLA129C
C                                                                       EPLA130C
  500 CONTINUE                                                          EPLA131C
      FYOVRX=-CFY*FY2                                                   EPLA132C
  505 DO 510 I=1,3                                                      EPLA133C
  510 SIG(I)=SEP(N,I)*OVRX                                              EPLA134C
  520 SIGZN= SGZI- DSIGZ3*OVRX                                          EPLA135C
      SGZTT=SIGZN                                                       EPLA136C
  525 CONTINUE                                                          EPLA137C
      DO 550 I=1,3                                                      EPLA138C
  550 SIG(I)=SIGP(I)-SIG(I)                                             EPLA139C
      STJ1=SIG(1)+SIG(2)+SIGZN                                          EPLA140C
      STJ2=((SIG(1)-SIG(2))**2+(SIG(2)-SIGZN)**2+                       EPLA141C
     1  (SIGZN-SIG(1))**2)/6.+SIG(3)**2                                 EPLA142C
      NST=NST+1                                                         EPLA143C
      IF (OVRX .EQ. 1 .AND. NST .EQ. 1) GO TO 310                       EPLA144C
      FYCUR=ALP*STJ1+SQRT(STJ2)-FK                                      EPLA145C
      IF (E(7,MTYPE) .LT. 0.0001) GO TO 555                             EPLA146C
      IF (OVRX .EQ. 1.) GO TO 552                                       EPLA147C
      IF (FYCUR .GE. FYCUR1) GO TO 555                                  EPLA148C
  552 FYCUR1=FYCUR                                                      EPLA149C
      IF (FYCUR .LT. FYOVRX) GO TO 555                                  EPLA150C
      IF(NANALY .GE. 1) GO TO 555                                       EPLA151C
      IF (OVRX .GE. 1.5) GO TO 555                                      EPLA152C
      OVRX=OVRX+0.1                                                     EPLA153C
      GO TO 505                                                         EPLA154C
  555 CONTINUE                                                          EPLA155C
      DO 560 I=1,3                                                      EPLA156C
  560 SEP(N,I)=SEP(N,I)*OVRX                                            EPLA157C
      PRINT 2000, FY(N),FY2,FY2RT,FYCUR,OVRX,SIGZT                      EPLA158C
      STRS(N,4)=SIGZN                                                   EPLA159C
 2000 FORMAT (22X,8E12.4)                                               EPLA160C
  570 FY(N)=FY2                                                         EPLA161C
  600 CONTINUE                                                          EPLA162C
      DO 650 I=1,3                                                      EPLA163C
  650 SEP(N,I)=SEP(N,I)+RSTRS(I)                                        EPLA164C
  700 CONTINUE                                                          EPLA165C
      RETURN                                                            EPLA166C
      END                                                               EPLA167C
```

```
      SUBROUTINE STRSTR (STJ1,STJ2,SIGZT,NEP,N,NCALC,NEXIT)
      COMMON /ELDATA/IX(900,5),MTAG(900),EPS(900),STRS(900,4),SEP(900,3)
      COMMON /MATP/ MTYPE,RO(12),E(8,12),AKO(12),MNTEN(12),MJNT(12)
     1         ,CRAC(12)
      COMMON /ARG/ RRR(5),ZZZ(5),S(10,10),P(10),RSTRS(4),LBAD,LM(4),
     1             ANGLE(4),XI,HH(6,10),C(4,4),EE(4),H(6,10),D(6,6),
     2             F(6,10),SIG(6),DSIG(6),RR(4),HSEL(31,4),DSIGZ
      DO 100 I=1,4
      DO 100 J=1,4
      C(I,J)=0.
  100 CONTINUE
      IF(NEP.EQ.1) GO TO 250
      IF (NCALC .LE. 2) GO TO 104
      IF(MTAG(N).GT.3) GO TO 200
  104 CONTINUE
      DO 105 KK=1,4
  105 EE(KK)=E(KK+1,MTYPE)
C
C**** MTAG=1 MEANS THAT BOTH PRINCIPAL STRESSES ARE COMPRESSIVE
C**** MTAG=2 MEANS THAT ONLY THE MAJOR PRINCIPAL STRESS IS TENSILE
C**** MTAG=3 MEANS THAT BOTH PRINCIPAL STRESSES ARE TENSILE
C**** MTAG=4 MEANS THAT THE ELEMENT FAILS IN COMPRESSION
C**** MTAG=5 MEANS THAT IT FAILS UNDER TENSILE MAJOR PRINCIPAL STRESS AN
C**** MTAG=6 MEANS THAT IT FAILS UNDER BOTH TENSILE PRINCIPAL STRESSES A
C
      IF(MTAG(N)-2) 182,184,183
  182 EE(3)=EE(1)
      GO TO 184
  183 EE(1)=EE(3)
  184 EE(1)=EE(1)/(1.0-EE(2)**2)
      EE(3)=EE(3)/(1.-EE(2)**2)
      EE(2)=EE(2)/(1.-EE(2))
      XX=EE(1)/EE(3)
      COMM=EE(1)/(XX-EE(2)**2)
      C(1,1)=COMM*XX
      C(1,2)=COMM*EE(2)
      C(2,1)=C(1,2)
      C(2,2)=COMM
      C(3,1)=C(1,2)
      C(3,2)=C(1,2)
      C(4,4)=.5*EE(1)/(XX+EE(2))
      GO TO 300
  200 DO 210 I=1,3
      SIG(I)=STRS(N,I)-SEP(N,I)
  210 CONTINUE
      SIGZT=STRS(N,4)
      STJ1=SIG(1)+SIG(2)+SIGZT
      STJ2=((SIG(1)-SIG(2))**2+(SIG(2)-SIGZT  )**2+(SIG(1)-SIGZT  )
     1**2)/6.0+SIG(3)**2
  250 SRJ2= SQRT(STJ2)
      IF (STJ2 .GT. 1.E-6) GO TO 270
      NEXIT=NEXIT+1
      PRINT 2000, N,STJ1,STJ2
 2000 FORMAT(10H *********,2X,*EL.NO.=*,I5,5X,*J1=*,E15.5,*J2=*,E15.5)
  270 CONTINUE
      PN=E(3,MTYPE)
```

```
      ALP=E(7,MTYPE)
      EK=E(6,MTYPE)
      EC=E(2,MTYPE)
      EG=EC/((1.+PN)*2.)
      EK=EC/(3.*(1.-2.*PN))
      HD=1.+9.*(ALP**2)*EK/EG
      H1=(1.5*EK*ALP/EG-STJ1/(6.*SRJ2))/(SRJ2*HD)
      H2=((ALP-STJ1/(6.*SRJ2))*(3.*EK*ALP/EG-STJ1/(3.*SRJ2))-
     1    3.*PN*EK*EK/(EC*SRJ2))/HD
      H3=0.5/(STJ2*HD)
C
C**** CALCULATE ELASTO-PLASTIC STRESS-STRAIN RELATIONSHIP
C
      C(1,1)=2.*EG*(1.-H2-2.*H1*SIG(1)-H3*SIG(1)**2)
      C(1,2)=-2.*EG*(H2+H1*(SIG(1)+SIG(2))+H3*SIG(1)*SIG(2))
      C(1,4)=-2.*EG*(H1*SIG(3)+H3*SIG(1)*SIG(3))
      C(2,1)=C(1,2)
      C(2,2)=2.*EG*(1.-H2-2.*H1*SIG(2)-H3*SIG(2)**2)
      C(2,4)=-2.*EG*(H1*SIG(3)+H3*SIG(2)*SIG(3))
      C(3,1)=-2.*EG*(H2+H1*(SIG(1)+SIGZT)+H3*SIG(1)*SIGZT)
      C(3,2)=-2.*EG*(H2+H1*(SIG(2)+SIGZT)+H3*SIG(2)*SIGZT)
      C(3,4)=-2.*EG*(H1*SIG(3)+H3*SIG(3)*SIGZT)
      C(4,1)=C(1,4)
      C(4,2)=C(2,4)
      C(4,4)=2.*EG*(0.5-H3*SIG(3)**2)
C
  300 IF(EPS(N).EQ.0.0) GO TO 400
C
      SS=SIN(EPS(N))
      CC=COS(EPS(N))
      S2=SS*SS
      C2=CC*CC
      SC=SS*CC
      D(1,1)=C2
      D(1,2)=S2
      D(1,3)=0.0
      D(1,4)=SC
      D(2,1)=S2
      D(2,2)=C2
      D(2,3)=0.0
      D(2,4)=-SC
      D(3,1)=0.0
      D(3,2)=0.0
      D(3,3)=1.0
      D(3,4)=0.0
      D(4,1)=-2.0*SC
      D(4,2)=-D(4,1)
      D(4,3)=0.0
      D(4,4)=C2-S2
      DO 350 II=1,4
      DO 350 JJ=1,4
      H(II,JJ)=0.0
      DO 350 KK=1,4
  350 H(II,JJ)=H(II,JJ) +C(II,KK)*D(KK,JJ)
      DO 360 II=1,4
      DO 360 JJ=1,4
```

```
      C(II,JJ)=0.0                                              STRS1130
      DO 360 KK=1,4                                             STRS1140
  360 C(II,JJ)=C(II,JJ)+D(KK,II)*H(KK,JJ)                       STRS1150
  400 CONTINUE                                                  STRS1160
      RETURN                                                    STRS1170
      END                                                       STRS1180
```

```
      SUBROUTINE NPFORC(NUMPC,IJBC,NPCAV,PSCA,NL,NM,KSHIFT,NNN,PRATIO,
     1   R,Z,B,NPCA,CODE)
      DIMENSION IJBC(50,2),PSCA(75,3),PSBC(6),PRATIO(20),R(999),
     1  Z(999),B(90),NPCA(75),CODE(999)
      DIMENSION ID(2)
      ID(1)=2
      ID(2)=1
C
C     BOUNDARY CONDITIONS
C
C       1. PRESSURE B.C.
C
      DO 300 L=1,NUMPC
      I=IJBC(L,1)
      J=IJBC(L,2)
      NTP=1
      IF ((I.GE.NL) .AND. (I.LE.NM)) NTP=0
      IF ((J.GE.NL) .AND. (J.LE.NM)) NTP=0
      IF (NTP .EQ. 1) GO TO 300
      DR=R(J)-R(I)
      DZ=Z(J)-Z(I)
      SINA=1.0
      COSA=0.0
      IF(ABS(DR) .LT. 1.E-10) GO TO 252
      AG=ATAN2(DZ,DR)
      SINA=SIN(AG)
      COSA=COS(AG)
  252 S2=SINA*SINA
      C2=COSA*COSA
      SC=SINA*COSA
      KD=0
      DO 253 NC=1,NPCAV
      IF (I .EQ. NPCA(NC)) KD=1
      IF (J .EQ. NPCA(NC)) KD=2
      IF (KD .EQ. 0) GO TO 253
      KDP2=KD+2
      KDP4=KD+4
      PSBC(KD)=PSCA(NC,1)
      PSBC(KDP2)=PSCA(NC,2)
      PSBC(KDP4)=PSCA(NC,3)
  253 KD=0
      DO 255 M=1,2
      SIGNN=S2*PSBC(M)+C2*PSBC(M+2)-2.*SC*PSBC(M+4)
      SIGT=-SC*PSBC(M)+SC*PSBC(M+2)+(C2-S2)*PSBC(M+4)
      PSBC(M)=SIGNN
      MP2=M+2
  255 PSBC(MP2)=SIGT
      DO 290 M=1,2
      N=ID(M)
      I=IJBC(L,M)
      J=IJBC(L,N)
      IF((I.LT.NL) .OR. (I.GT.NM) ) GO TO 290
      I2=2*I-KSHIFT
      I1=I2-1
      PI=PSBC(M)
      PJ=PSBC(N)
```

```
      SI=PSRC(M+2)
      SJ=PSRC(N+2)
      PM=(2.*PI+PJ)/6.
      SM=(2.*SI+SJ)/6.
      R1=-DZ*PM+DR*SM
      R2= DR*PM+DZ*SM
      R1=R1*PRATIO(NNN)
      R2=R2*PRATIO(NNN)
      SINA=0.0
      COSA=1.0
      IF (CODE(I) .GE. 0.) GO TO 280
      AG=CODE(I)/57.29577
      SINA=SIN(AG)
      COSA=COS(AG)
  280 B(I1)=B(I1)+R1*COSA+R2*SINA
      B(I2)=B(I2)-R1*SINA+R2*COSA
  290 CONTINUE
  300 CONTINUE
  310 CONTINUE
      RETURN
      END
```